Name of the Course : **Generic Elective**

Unique Paper Code : **32355301**

Name of the Paper : **GE-3 Differential Equations**

Semester : **III**

Duration : **3 Hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. (i) Solve

$$xdy - ydx = \sqrt{x^2 + y^2}\,dx.$$

(ii) Solve the initial value problem

$$\left(x^2 + y^2 + x\right)dx + xy\,dy = 0, \qquad y(1) = 1.$$

(iii) Solve the initial value problem

$$x\frac{dy}{dx} + y = y^2 \log x, \qquad y(1) = -1.$$

2. (i) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, a being parameter of the family.

(ii) Find a family of oblique trajectories that intersect the family of circles $x^2 + y^2 = c^2$ at angle 45°.

(iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}.$$

3. (i) Solve by method of variation of parameters

$$y'' + y = \operatorname{cosec} x.$$

(ii) Solve by method of undetermined coefficients

$$y'' + 1.44y = 24\cos 1.2\,x.$$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0,\ y(0) = -1,\ y'(0) = 30,\ y''(0) = 28.$$

4. (i) Show that $\{e^{-x}, e^{3x}, e^{4x}\}$ forms a basis of the solution set of the equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0.$$

(ii) Solve the initial value problem

$$x^2\, y'' - 2x\, y' - 10y = 0,\ \ y(1) = 5,\ y'(1) = 4.$$

(iii) Solve the linear system

$$y_1' = 2y_1 + 5y_2$$
$$y_2' = 5y_1 + 12.5y_2$$

5. (i) Find the partial differential equation arising from the surface

$$z = xy + f\left(x^2 + y^2\right).$$

(ii) Find the general solution of the partial differential equation

$$u_x + 2xy^2\, u_y = 0.$$

(iii) Apply the method of separation of variables $u(x, y) = f(x)\, g(y)$ to solve

$$y u_x + x u_y = 0 \quad \text{on} \quad u(0, y) = y^2.$$

6. Reduce each of the following equations into canonical form and find the general solution:

(i) $u_x - u_y = u, \quad u(x, 0) = 4e^{-3x}.$

(ii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3y u_y = 0.$

(iii) $u_{xx} - 3u_{xy} + 2u_{yy} = 0.$

Name of the Course : CBCS-(LOCF)-Generic Elective B.A.(Prog.)/ B.Com(Prog)

Unique Paper Code : 62355503

Name of the Paper : GE- General Mathematics-I

Semester : V

Duration : 3 hours

Maximum Marks : 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Write a short note on the life and mathematical contribution of any of three of the following Mathematicians:
   a. Aryabhata
   b. Bhaskara- II
   c. Paramesvara
   d. Brahmagupta

2. Define Perfect numbers and Amicable numbers. State the properties of Perfect numbers.

   Define unit fraction and express $\frac{3}{4}$ and $\frac{5}{8}$ as unit fraction.
   Define algebraic numbers and transcendental numbers. Why π is not an algebraic number?

3. Define the Inversion and explain The Fifteen Puzzle.

   Find the remainder when

   $12345 \times 123456 \times 1234567$ $is\ divided\ by$ 13 .

   What is the Euclidean algorithm? Find the greatest common divisor of 60 and 25.

4. Find the number of distinct permutations of the letters in "Karnataka" and "Chennai"?

   Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 3 \\ 2 & 1 & 4 \end{bmatrix}$. Compute the product AB and BA whichever exists.

5. Express the matrix A as the sum of a symmetric and skew symmetric matrix

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$$

Let $C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find λ so that $C^2 = 5C + \lambda I$

6. Use Cramer's Rule to solve for x and y in the below two equations

$$x - 2y = 4$$
$$-3x + 5y = -7$$

If $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, Find det(AB), det(A) and det(B) and Verify

whether det(AB) = det(A) * det(B)

Name of Course	: **B.A.(Prog.) DSE : Mathematics**
Unique Paper Code	: **62357503**
Name of Paper	: **DSC- Statistics**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs.

i. Suppose that three bulbs are randomly selected. What is the probability that exactly two of the selected bulbs are rated 75 W?

ii. If two bulbs are randomly selected from the box of lightbulbs and at least one of them is found to be rated 75 W, what is the probability that both of them are 75-W bulbs?

iii. Given that at least one of the two selected is not rated 75 W, what is the probability that both selected bulbs have the same rating?

Suppose A and B are independent events. Show that the following pairs of events are also independent:

i. A and $\bar{B}$

ii. $\bar{A}$ and $\bar{B}$

2. Evaluate the first four moments about the mean for the random variable with probability density function

$$f(x) = \begin{cases} \frac{4x}{81}(9 - x^2), & 0 \leq x \leq 3 \\ 0, & elsewhere \end{cases}$$

A and B throw with one die for a prize of Rs. 11 which is to be won by the player who first throws 6. If A has the first throw, what are their respective expectations?

3. Suppose that flaws in plywood occur at random with an average of one flaw per 50 square feet. What is the probability that a 4×8 square feet sheet will

i. have no flaws

ii. atmost one flaw

(Assume that number of flaws per unit area is Poison distributed.)

Suppose that a trainee soldier shoots a target in an independent manner. If the probability that the target is hit on any one shot is 0.8, what is the probability that the target would be hit

i. on the sixth attempt
ii. in fewer than 5 shots
iii. in even number of shots.

4. Consider a sample of size 2 drawn without replacement from an urn containing three balls numbered 1, 2 and 3. let X be the number on the first ball drawn and Y the larger of the two numbers drawn.

i. Find the joint discrete density function of X and Y
ii. $P(X = 1/Y = 3)$
iii. Find the correlation coefficient of X and Y

The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{x+y}{3} & 0 \le x \le 1, 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

Find the two lines of regression.

5. Let $(X_k), k = 1,2, \dots \dots$ be a sequence of independent random variables defined as

$P\left(X_k = \frac{1}{\sqrt{k}}\right) = \frac{2}{3}$ and $P\left(X_k = -\frac{1}{\sqrt{k}}\right) = \frac{1}{3}$.

Examine if the Weak Law of Large Numbers holds for this sequence.

Two independent samples of 8 and 7 items respectively have the following values of the variables:

Sample 1: 9 11 13 11 15 9 12 14

Sample 2: 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly? Given that for (7,6) degrees of freedom the value of F at 5% level of significance is 4.21.

6. In a random sample of 500 men from a particular district of UP, 300 are found to be smokers. In one of 1000 men from another district, 550 are smokers. Do the data indicate that the two districts are significantly different with respect to the prevalence of smoking among the men.

In experiments on pea-breeding, Mendel got the following frequencies of seeds: 315 round and yellow, 101 wrinkled and yellow, 108 round and green, 32 wrinkled and green: total 556. Theory predicts that the frequencies should be in the proportion 9:3:3:1. Examine the correspondence between theory and experiment. (chi-square at 3 degrees of freedom at 5% level of significance is 7.815)

Name of the Course : **B.A.(Prog.)**

Semester : **V**

Unique Paper Code : **62353606**

Name of the Paper : **SEC-3: Transportation and Network Flow Problems**

Duration: **2 Hours** Maximum Marks: 55

Attempt any Four questions. All questions carry equal marks.

1. MG Auto has three plants in Delhi, Mumbai and Bangalore and four distribution centres at Goa, Chennai , Haryana and Kolkata. The capacities of the three plants during the next quarter are 500,600 and 250 cars respectively. The quarterly demands at the four distribution centres are 600, 400, 200 and 150 cars respectively. The transportation cost per car on the different routes is given in the table:

	Distribution Center			
Factory	Goa	Chennai	Haryana	Kolkata
Delhi	3	2	7	6
Mumbai	7	5	2	3
Bangalore	2	5	4	5

(a) Compare the starting solution (Initial basic feasible solution) obtained by the North-West Corner Method & Least-Cost Method for the above transportation problem.

(b) Find the starting solution (Initial basic feasible solution) using Vogel Approximation Method (VAM) and hence find the optimal solution by the method of multipliers.

2. Solve the following cost minimization assignment problem:

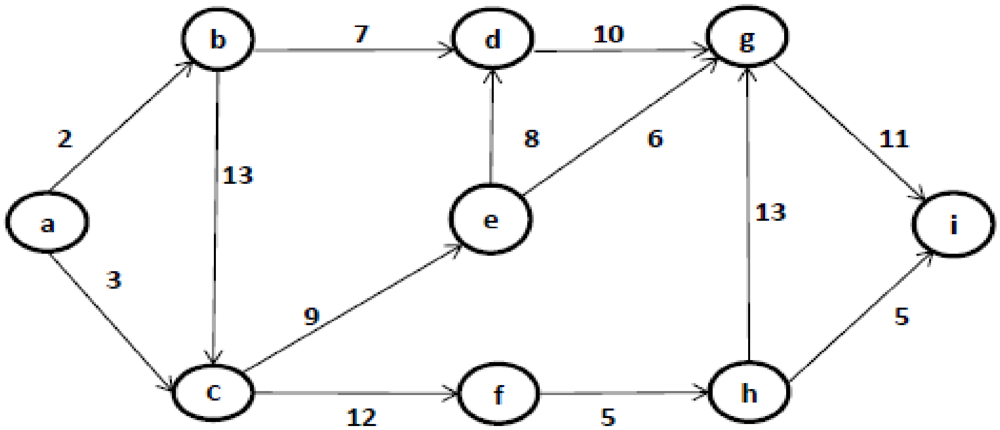
	Men				
Jobs	**I**	**II**	**III**	**IV**	**V**
A	2	3	5	5	6
B	4	5	7	7	8
C	7	8	8	10	9
D	3	5	3	6	5
E	4	3	5	2	1

Does this problem has more than one solution? If yes, then find any two possible solutions.

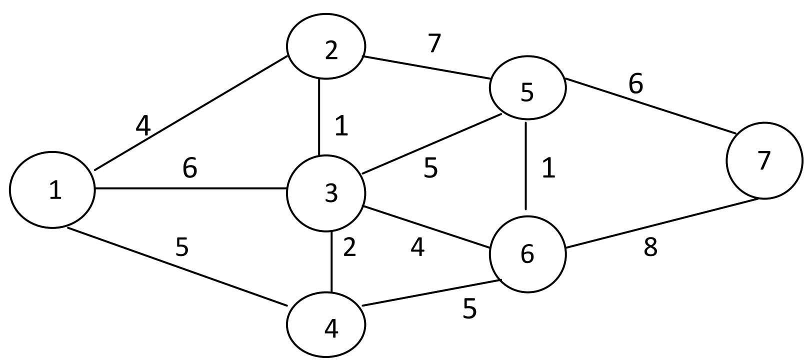
3. Solve the following travelling-salesman problem:

	To city				
From city	**I**	**II**	**III**	**IV**	**V**
I	M	5	8	4	5
II	5	M	7	4	5
III	8	7	M	8	6
IV	4	4	8	M	8
V	5	5	6	8	M

4. Use Dijkstra's algorithm to find the shortest path from source '**a**' to destination '**i**' from the following network:



5. The Midwest TV Cable company is in the process of providing cable services to new six housing development areas. The adjoining figure depicts possible T V linkages among the six areas. The cable miles are shown on each arc. Determine the most economical cable network for the company starting at node 7. Also determine the minimal spanning tree starting at node1 for the given network when Nodes 3 and 5 are linked by 3 miles and Node 2 cannot be linked directly to node 3.



6. The activities associated with a certain project are given below:

Activity	Predecessor Activity	Duration (Weeks)
A	--	4
B	--	3
C	A, B	2
D	A, B	5
E	B	6
F	C	4
G	D	3
H	F, G	7
I	F, G	4
J	E, H	3

Develop the Associated Network for the project and find the minimum time of completion of the project. Also determine a critical path and critical activities for the project network. Find Early start time and Latest finish time of each activity.

Name of Course	: **CBCS B.A. (Prog.)**
Unique Paper Code	: **62353327_LOCF**
Name of Paper	: **SEC-Computer Algebra System**
Semester	: **III**
Duration	: **2 hours**
Maximum Marks	: **38 Marks**

Attempt any four questions. All questions carry equal marks.

Q1. Form the following grid using appropriate functions

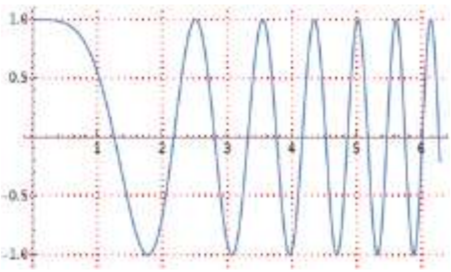
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625

Each column is right aligned.

Graph each of the functions

a. $g(x, y) = 19\cos(x) + \sin(\sqrt{7}\, x)$, $-20 \le x \le 20$
b. $2x + 1,\ 1 - x^2,\ 1 - x - x^2/3$, $-2 \le x \le 2$

Q2.



Create the above plot of $f(x)=\cos(x^2)$, $0 \le x \le 2\pi$, with thick, red and dotted grid lines. Change the code so that the grid lines appear only at $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and y=.1,.3,.5,.7,.9

Compute each expression.
a. $3^8 \bmod 7$
b. $5^{10} \bmod 11$

Q3. Write the command to define a polynomial

$$f(x) = x(x^2 + y^2) - a(x^2 - y^2)$$

Plot it in $-a \le x \le a$. Write a syntax to solve

$$g(x) = 3x^3 - 16x^2 + 23x - 6$$

and find the numerical solution of $g(x) = 0$.

Q4. Give the syntax to find the derivative of $f(x) = \frac{1}{(1+4x^2)\sqrt{1+3x^2}}$ with respect to x, then evaluate

$$\int_0^\infty \frac{1}{(1+4x^2)\sqrt{1+3x^2}}\,dx$$

and write a command to compute:

$$\sum_{p=0}^{100} (p+2)^5$$

Q5. Find the minimum value of the function $f(x) = x^{2/3}$and write syntax for it also. Find the extreme point of the function $g(t) = 9t^2 + 2t + 4$ and also plot the given function.

Q6. Generate a 9×9 matrix whose i, j th entry is i^2+2ij by using Table command. Also write syntax to get the fifth column of the matrix. Find the trace of the matrix.

Unique Paper Code : **62354343**

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : **Analytic Geometry and Applied Algebra**

Semester : **III (CBCS)**

Time : 3 Hours **Maximum Marks :75**

- **Attempt any four questions in all.**
- **All questions carry equal marks.**

1) (a) Writing the basic steps, describe and draw the graph of the given equation showing their vertices, foci and asymptotes.

$$4(y-3)^2 - 9(x-2)^2 = 36$$

(b) Identify and sketch the curve :

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

(c) Find an equation of the parabola that has its vertex at (1, 1) and directrix y = – 2. Also, state the reflection property of parabola.

2) (a) Describe, sketch and label the focus, vertex and directrix of the parabola

$$2y^2 - 6y - 3x + 4 = 0$$

(b) Find the equation of the ellipse whose axes are along the coordinate axes, vertices are (± 5, 0) and foci at (± 4, 0).

(c) Find an equation for hyperbola that has same foci as the ellipse

$$12x^2 + 16y^2 = 48$$

and asymptotes $y = \pm\frac{2x}{3}$

3) (a) Find an equation of the largest sphere contained in the cube determined by the planes x=2,x=16;y=4,y=18; and z=7,z=21.

(b) Rotate the coordinate axes to remove the xy-terms of the conic:

$$5x^2 - 6xy + 5y^2 - 49 = 0$$

Then name the conic.

(c) (i) Find the vector of length 4 that makes an angle $\pi/6$ with positive x-axis.

(ii) Find the angle between the vectors $\vec{u} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{v} = 2\hat{i} + 7\hat{j} + 6\hat{k}$.

4) (a) Find the equation of sphere that has (1, –2, 4) and (3, 4, –12) as end points of diameter.

(b) Given $\|\vec{a}\| = 10$, $\|\vec{b}\| = 2$ and $\vec{a} \cdot \vec{b} = 12$, find $\|\vec{a} \times \vec{b}\|$

(c) (i) Using vectors find the area of triangle with vertices A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1).

(ii) Use a scalar triple product to determine whether the vectors $\vec{u} = 4\hat{i} - 8\hat{j} - \hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{w} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ lie in the same plane.

5) (a) Show that the lines:

$$L_1 : x = 1 + 4t, \quad y = 5 - 4t, \quad z = -1 + 5t$$
$$L_2 : x = 2 + 8t, \quad y = 4 - 3t, \quad z = 5 + t$$

Are skew lines and find the distance between them.

(b) (i) Find the distance between the point (1, –4, –3) and the plane

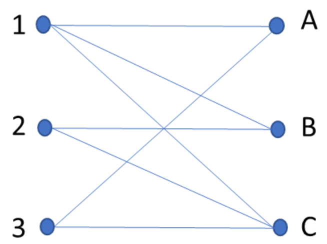
$$2x - 3y + 6z = -1$$

(ii) Determine whether the line:
$L : x = 3 + 8t,\ y = 4 + 5t,\ z = -3 - t$ is parallel to the plane $x - 3y + 5z = 12$.

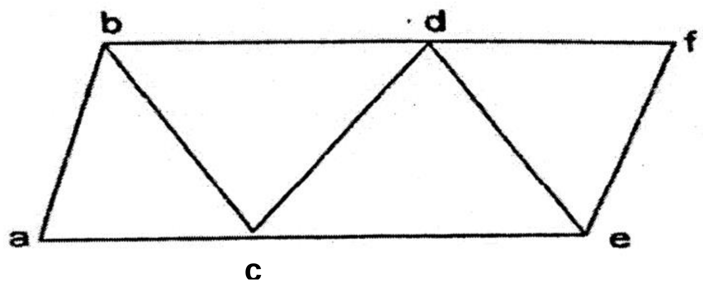
(c) Find the volume of the tetrahedron with vertices P(1, 2, 0), Q(2, 1, 3), R(–1,0, 1) and S(3, –2, 3)

6) (a) A supermarket wishes to test the effect of putting cereal on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of cereal.

(b) Find a matching for the following graph or explain why none exists



(c) Find a minimal edge cover for the following graph. Given a detailed logical analysis.


b
d
f
a
c
e

Unique Paper Code : 62354343

Name of the Course : B.A. (Prog.) Mathematics

Name of the Paper : Analytic Geometry and Applied Algebra

Semester : III (CBCS)

Time : 3 Hours **Maximum Marks :75**

- **Attempt any four questions in all.**
- **All questions carry equal marks.**

1. (a) Identify and sketch the curve:

$$y = 4x^2 + 8x + 5$$

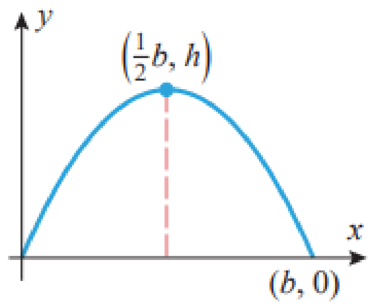
Also label the focus, vertex and directrix

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



2. (a) Find the equation of parabola with vertex (2,4) and focus (3,4).

(b) Find the equation for the ellipse that has ends of major axis (±6, 0) and passes through (2, 3)

(c) Find the equation for a hyperbola that satisfies the given conditions:
Asymptotes y = 2x + 1, y = –2x + 3 and passes through the origin.

3. (a) Find an equation of the sphere with centre (2,-1,-3) and satisfying
   i) Tangent to the x-y plane.

ii) Tangent to the x-z plane.
iii) Tangent to the y-z plane.

b) Show that the graph of the equation:

$$\sqrt{x} + \sqrt{y} = 1, \quad \forall x \in [0,1], y \in [0,1]$$

is a portion of a parabola.

(c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

4. (a) Find $\vec{u}$ and $\vec{v}$ if $5\vec{u} + 2\vec{v} = 6\hat{i} - 5\hat{j} + 4\hat{k}$ and $3\vec{u} - 4\vec{v} = \hat{i} + 2\hat{j} + 9\hat{k}$ Also find a vector of length 3 and oppositely directed to $\vec{v}$.

(b) (i) Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

(ii) Determine whether $\vec{u} = <6, 1, 3>$ and $\vec{v} = <4, -6, -7>$ make an acute angle, an obtuse angle or are orthogonal? Justify your answer.

(c) Find the volume of the parallelopiped with adjacent edges $\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$. Also find the area of the face determined by $\vec{u}$ and $\vec{v}$.

5. (a) Find the distance of the point P(2, 5, –3) from the plane

$$\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$$

(b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(1, 0, -3)$ that is perpendicular to the plane 4x + y + 3z = 2.

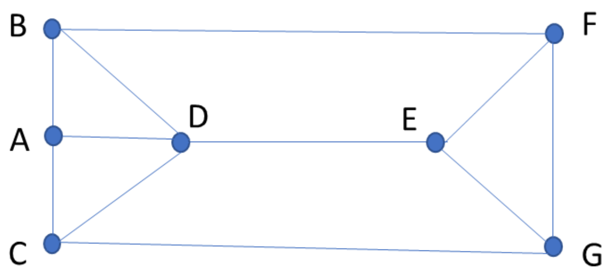
(c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$$L_1 : x = 2 - t, \qquad y = 2t, \qquad z = 3 + t$$

$$L_2 : x = -1 + 2t, \qquad y = 3 - 4t, \qquad z = 5 - 2t$$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.