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S. No. of Question Paper : 8525

Unique Paper Code : 32355101

J

Name of the Paper : Calculus

Name of the Course : Mathematics : G.E. for Honours

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts from each question.

I. (a) Find the open interval on which $f(x) = x^3 - 3x + 3$ is concave up and concave down. Also determine points of inflection, if any.

(b) Find the interval in which the function $f(x)$ is (i) increasing
(ii) decreasing $f(x) = 2x^3 - 9x^2 + 12x$.

(c) Evaluate :

$$\lim_{x \rightarrow -\infty} \frac{3x^2 - 5x + 7}{7x^2 + 2x - 3}$$

6+6

P.T.O.

2. (a) Find the volume of the solid that results when the region enclosed by $y = x^2$, $x = 0$, $x = 2$, $y = 0$, is revolved about x -axis.
- (b) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^2$, $x = 1$, $x = 4$, $y = 0$ is revolved about y -axis.
- (c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$, $x = 2$ about y -axis. 6/11

3. (a) Evaluate :

$$\lim_{x \rightarrow 0} \frac{10(\sin x - x)}{x^3}.$$

- (b) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

- (c) Find the asymptotes of the graph of the function :

$$f(x) = -\frac{8}{x^2 - 4}.$$

4. (a) Identify the symmetries of the curve $r^2 = \cos \theta$ and then sketch the curve.

- (b) Solve the initial value problem and find \vec{r} as a vector valued function of t .

$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{\frac{1}{2}}\hat{i} + e^{-1}\hat{j} + \frac{1}{t+1}\hat{k}, \quad \vec{r}(0) = \hat{k}.$$

- (c) Find a unit tangent and unit normal vector for space curve :

$$\vec{r}(t) = (\cos t + t \sin t)\hat{i} + (\sin t - t \cos t)\hat{j} + 3\hat{k}. \quad 6+6$$

5. (a) Write acceleration \vec{a} in the form $a_T T + a_N N$ without finding T and N for :

$$\vec{r}(t) = (t+1)\hat{i} + 2t\hat{j} + t^2\hat{k} \text{ at } t=1.$$

- (b) Show that :

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & \end{cases}$$

is continuous at every point except at origin.

P.T.O.

(4)

(c) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$, where $w = x + 2y + z^2$, x
 $y = r^2 + \ln s$, $z = 2r$.

6. (a) Find the direction in which $f(x, y) = xe^y + z^2$

(i) increases most rapidly at $P\left(1, \ln 2, \frac{1}{2}\right)$

(ii) decreases most rapidly $P\left(1, \ln 2, \frac{1}{2}\right)$.

(b) Find equations of tangent plane and normal lines for
curve $z^2 - 2x^2 - 2y^2 - 12 = 0$ at $P(1, -1, 4)$.

(c) Find all the local maxima, local minima and saddle points
of the curve :

$$f(x, y) = 4xy - x^4 - y^4. \quad 6\frac{1}{2}$$

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S. No. of Question Paper : 7168

Unique Paper Code : 62357502 J

Name of the Paper : Differential Equations

Name of the Course : B.A. (Prog.) Mathematics : DSE-2

Semester : V

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

1. (a) Solve the initial value problem :

6

$$(y + \sqrt{x^2 + y^2}) dx - x dy = 0; \quad y(1) = 0.$$

(b) Solve :

6

$$(xy^2 - x^2) dx + (3x^2y^2 + x^2y - 2x^3 + y^2) dy = 0$$

(c) Solve :

6

$$y + px = x^4 p^2$$

2. (a) Solve :

6.5

$$\frac{d^2y}{dx^2} + 4 = \cos 2x + \sin 2x$$

(b) Solve :

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10(x + \frac{1}{x})$$

(c) Consider the differential equation :

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$$

(i) Verify that $y_1 = e^x$ and $y_2 = e^{2x}$ are the sol
of the above differential equation.

(ii) Find a particular solution of the form

$$y = c_1 y_1 + c_2 y_2$$

that satisfies the initial condition $y(0) =$
 $y'(0) = 0$.

3. (a) Using the method of variation of parameters, solve

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \log x, \quad (x > 0)$$

(b) Given that $y = x + 1$ is a solution of different
equation :

$$(x+1)^2 \frac{d^2 y}{dx^2} - 3(x+1) \frac{dy}{dx} + 3y = 0$$

Find a linearly independent solution by reducing the
order and write the general solution.

(c) Solve :

6

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 2y = x^2 \log x + 3x$$

4. (a) Solve the following system of equations :

6.5

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,$$

$$\frac{dy}{dt} + 3y + 5x = 0.$$

(b) Solve :

6.5

$$\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{5z + \tan(y - 2x)}.$$

(c) Solve :

6.5

$$(ydx + xdy)(x - z) + xydz = 0.$$

5. (a) Eliminate the arbitrary function f from the equation : 6

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

(b) Find the general solution of the differential equation : 6

$$x(y^2 - z^2)q - y(x^2 + z^2)p = (x^2 + y^2)z.$$

(c) Find the complete integral of the partial differential equation : 6

$$16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0.$$

P.T.O.

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic :

$$x(xy-1)r - (x^2y^2-1)s + y(xy-1)t + (x^2+y^2-1)p + (y-1)q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

(ii) Form a partial differential equation by eliminating constants a , b from the relation :

$$z = ax + by + cxy.$$

(b) Find the general solution of the differential equation

$$x^2(y-x)q + y^2(x-y)p = z(x^2 + y^2).$$

(c) Find the complete integral of

$$px + qy = pq.$$



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Roll No.

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S. No. of Question Paper : 7280

Unique Paper Code : 42353327 J

Name of the Paper : Mathematical Typesetting System

Latex

Name of the Course : B.Sc. (Prog.)/B.Sc. Math. SC : SEC

Semester : III

Duration : 2 Hours Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(i) Boldfaced mathematical text is produced with a command.

(ii) The symbols may be used instead of pair of \$ sign.

(iii) Emphasized text is produced with command.

(iv) For plotting a function with PSTricks, we need the package.

P.T.O.

(v) Output of the command $\$\\frac{d}{dx}\\left(int_0^x f(t)dt\\right)=f(x) \$$ is

2. Answer any six parts from the following : $2 \cdot 5 \times 6 = 15$

(i) Write the command in LaTeX to obtain the expression :

$$\left(\frac{a+b}{x+y} \right)^{\frac{2}{3}}$$

(ii) Write the command in LaTeX to obtain the expression :

$$(a + b + a^2b + ab^2)^2.$$

(iii) Explain the command `\pscircle(3,2.5){2.5}`.

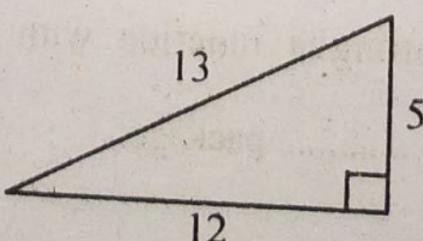
(iv) Explain the command `\psarc(1,1){2}{0}{65}`.

(v) What is the difference between the commands `\eqnarray` and `\eqnarray*`.

(vi) Write a code in LaTeX to get the following :

$$\begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

(vii) Write the command in PSTricks to draw the following picture :



(viii) Write the command in PSTRicks to plot the function $y = \sin x$.

(ix) What is wrong with the following input :

If $\$theta = pi$, then $\$sin theta = 0$$$

3. Answer any four parts from the following : $4 \cdot 5 \times 4 = 18$

(i) Using beamer produce a presentation with the following content :

Slide 1 : Title of the presentation with authors name and

date

Slide 2 : Some trigonometry identities :

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$2 \sin \theta \cos \theta = \sin 2\theta$$

Slide 3 : Thank you

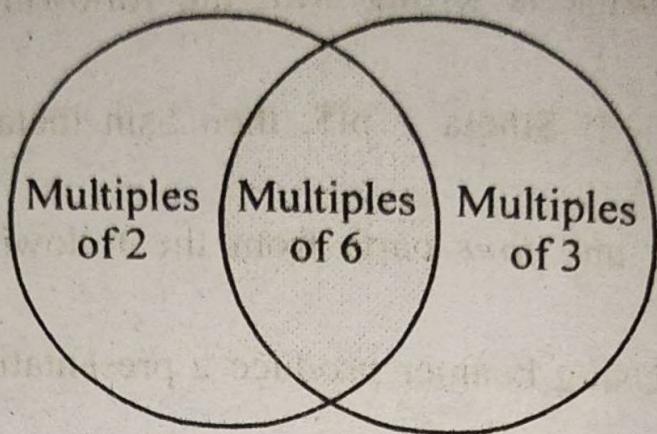
(ii) Write the code in LaTeX to typeset the following matrix :

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$



(4)

- 7280
(iii) Write the code in PStricks to draw the following picture :



- (iv) Write a code in LaTeX to get the following :

The general solution to the differential equation
 $y'' - 3y' + 2y = 0$ is

$$y = c_1 e^x + c_2 e^{2x}$$

- (v) Write a code in LaTeX to get the following :

$$|x - 2| = \begin{cases} x - 2 & x \geq 2 \\ -x + 2 & x < 2 \end{cases}$$



This question paper contains 5 printed pages.

Your Roll No.

J

Sl. No. of Ques. Paper : 7289

Unique Paper Code : 42353503

Name of Paper : Statistical Software R

Name of Course : B.Sc. (Math. Sc.) / B.Sc. (Prog.) :
SEC

Semester : V

Duration : 2 hours

Maximum Marks : 38

(Write your Roll No. on the top immediately
on receipt of this question paper.)

All questions are compulsory.

All commands should be written in software R.

1. Do any five of the following:

State whether the following statements are true or false:

- (a) The commands mean() and rowMeans() for a data frame give the same output.
- (b) read.csv(choose.file()) is used to read a file.
- (c) If we don't have any named object at all, then ls() command gives NA.
- (d) getwd() and setwd() are same commands.
- (e) \$ syntax is used to copy a data.

P. T. O.

(f) `runif(10)` creates ten random numbers.

1x5

2. Do any five of the following:

Fill in the blanks:

- (a) For rotating data tables, we can use the command (`trans()`, `t()`).
- (b) command produces the sum values for rows. (`rowsums()` / `rowSums()`)
- (c) `hist()` command is used for (history, histogram).
- (d) command can sort the data. (`order()` / `rank()`)
- (e) command can be used to save one or more objects to a file. (`load()` / `save()`)
- (g) Tables can be summarized using the command. (`apply()` / `attach()`)

1x5

3. Write the commands for the following:

- (a) (i) Using `scan` command, enter the following data:

Week: sun, mon, tue, wed

- (ii) Insert the items thu, fri, sat at the end of the above vector.

- (b) (i) Write the command to print the object starting with 'ti'.

- (ii) Find the length of the vector `data_new`.
- (c) For the vector
 production: 9, 10, 15, 10, 6, 8, 11, NA
- (i) Find the largest of the given data, after removing the effect of NA.
 - (ii) Show the last three items of the vector.
- (d) (i) Differentiate between data structure and a matrix.
 (ii) Convert the following data into integer:
 Data 7: 23.0 17.5 14.5 12.3 12.9

Consider the following data, for the questions (e) - (h):

Len : 12, 23, 45, 23, 16, 31

Speed : 12, 34, 16, 21

Noo : 2, 6, 5, 8

- (e) Create a matrix mew.
- (f) (i) Display the data for the columns Len, Speed.
 (ii) Display the data of first and second row for the above sample.
- (g) (i) Convert the above matrix into a data frame.
 (ii) Convert the matrix into a list.
- (h) Add the appropriate row heads.

$$2 \times 8 = 16$$

P. T. O.



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4. Do any four of the following:

(a) (i) Create the following matrix:

>bird

	<i>Garden</i>	<i>Hedgerow</i>	<i>Parkland</i>
Blackbird	47	10	40
Chaffinch	19	3	5
Great Tit	50	0	10
Robin	9	3	8

(ii) Display the mean of second row and second column.

(iii) Display the sum of rows and columns of the matrix.

(b) (i) Make the data objects:

data1 : 3 3 8 4 2 7 1

data2 : a b c d e f g h

(ii) Create a Cleveland dot plot using above data. Set the background colour for the plotting symbols.

(iii) Set the character expansion factor for points and label the axes.

(c) Create mathematics1data file where,

Mathematics1 = 2, 9, 8, 4, 6, 2, 7, 5, 2, 7.

Also make a quantile-quantile plot.

(d) (i) Display the data frame:

> orchid

	closed	open
a	3	5
b	5	3
c	7	8
d	9	4
e	3	9

(ii) Scatter plot the columns of above data. Also, write down the command to use different plotting characters.

(iii) How do you get the best size and scale of each axis to fit the plotting area?

(e) (i) Make a vector:

Data2 : 3 3 8 4 2 7 1 5 7 2 8 7

(ii) Create a histogram for above data.

(iii) Specify the breaks of bars at nos. 2, 5, 6, 9.

(iv) Color the bars and suppress the main title for the histogram.

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Your Roll No. :

Sl. No. of Q. Paper : 7058 J

Unique Paper Code : 62351101- OC

Name of the Course : B.A. (Prog.)
Mathematics

Name of the Paper : Calculus

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates :

(a) Write your Roll No. on the top immediately
on receipt of this question paper.

(b) Attempt any two parts from each question.

1. (a) Discuss the existence of the limit of the
function

$$f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$$

as $x \rightarrow 0$.

6

(b) Examine the continuity of the function

$$f(x) = |x-2| + |x-3|$$

at $x = 2$ and $x = 3$.

6

P.T.O.

(c) Discuss the derivability of the function.

$$f(x) = \begin{cases} 2x - 3 & \text{if } 0 \leq x \leq 2 \\ x^2 - 3 & \text{if } 2 < x \leq 4 \end{cases}$$

at $x = 2$.

6

2. (a) If $y = \sin(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n \quad 6.5$$

$$(b) \text{ If } u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{y}{x}$$

where $x \neq 0$ and $y \neq 0$, prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}. \quad 6.5$$

(c) State Euler's theorem and using it prove that

$$\text{if } z = \log \left(\frac{x^4 + y^4}{x + y} \right) \text{ then } z \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3. \quad 6.5$$

3. (a) Show that the length of the portion of the tangent to the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between coordinate axes is constant. 6

(b) Find the equation of normal to the curve $y^2(x+a) = x^2(3a-x)$ at the point where $x = a$.

6

2

- c) Find the radius of curvature at any point
 $P(x, y)$ on the curve $x = e^t \cos t, y = e^t \sin t.$

6

- (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$

6.5

- (b) Find the position and nature of the double points on the curve

$$x^3 - 2x^2 - y^2 + x + 4y - 4 = 0$$

6.5

- (c) Trace the curve

$$x(x^2 + y^2) = a(x^2 - y^2)$$

6.5

- (a) Verify Lagrange's Mean Value Theorem for the function

$$f(x) = (x-1)(x-2)(x-3) \text{ in } [1, 4].$$

6

- (b) Separate the intervals in which the following function is increasing or decreasing : 6

~~$$f(x) = 2x^3 - 15x^2 + 36x + 1$$~~

- (c) Show that $\frac{x}{1+x} < \log(1+x) < x \quad \forall x > 0.$ 6

3

P.T.O.

6. (a) Find the maximum and minimum value of the function $f(x) = (x-1)(x-2)(x-3)$ 6.5
- (b) Obtain Maclaurin's series expansion for the function $f(x) = e^x$ for all $x \in \mathbb{R}$. 6.5
- (c) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$. 6.5



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This question paper contains 4 printed pages]

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S. No. of Question Paper : 8649

J

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : B.A. (Programme) Mathematics

Semester : I

Maximum Marks : 75

Duration : 3 Hours

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

Marks are indicated against each question.

I. (a) Evaluate : $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$. 6

(b) Examine for points of discontinuity of the function f defined on $[0, 1]$ as follows :

$$f(x) = \begin{cases} 0, & \text{if } x = 0 \\ \frac{1}{2} - x, & \text{if } 0 < x < \frac{1}{2} \\ \frac{1}{2}, & \text{if } x = \frac{1}{2} \\ \frac{3}{2} - x, & \text{if } \frac{1}{2} < x < 1 \\ 1, & \text{if } x = 1 \end{cases}$$

State the types of discontinuity also. 6

P.T.O.

(c) Discuss the derivability of the function :

$$f(x) = \begin{cases} -x^2, & \text{if } x \leq 0 \\ 5x - 4, & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$$

at $x = 0, 1, 2$.

6

2. (a) Show that $y = x + \tan x$ satisfies the differential equation

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0. \quad 6.5$$

(b) If $y = \cos(m \sin^{-1} x)$, show that : 6,5

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

(c) If $u = \sin^{-1} \frac{x+y}{\sqrt{x+y}}$, use Euler's theorem to show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u. \quad 6.5$$

3. (a) Find the point on the curve :

$$x = a(\theta + \sin \theta), y = a(1 - \cos \theta),$$

where the tangent is perpendicular to x-axis. 6

(b) Find the equation of the normal at (a, a) to the curve : 6

$$x^2y^3 = a^5.$$



- (c) Find the radius of curvature at any point of the curve : 6

$$x = a(\cot t + t \sin t), y = a(\sin t - t \cos t)$$

- (a) Find the asymptotes of the following curve : 6,5

$$xy^3 - x^3 = a(x^2 + y^2).$$

- (b) Find the position and nature of the double points on the curve : 6,5

$$y^2 = 2x^2y + x^4y - 2x^4.$$

- (c) Trace the curve : 6,5

$$9ay^2 = x(x - 3a)^2.$$

- (a) Show that there is no real number 't' for which the equation $x^2 - 3x + t = 0$ has two distinct roots in $[0, 1]$. 6

- (b) Show that : 6

$$\frac{x}{1+x} < \log(1+x) < x \text{ for all } x > 0.$$

- (c) State Lagrange's mean value theorem. Explain why Lagrange's mean value theorem is not applicable to the following function :

$$f(x) = \begin{cases} x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

for $x \in [-1, 1]$. 6



6. (a) Assuming the validity of expansion, find the Maclaurin's series expansion of $e^x \cos x$. 6.5

(b) Determine the values of p and q for which

$$\lim_{x \rightarrow 0} \frac{x(1 + p \cos x) - q \sin x}{x^3} \text{ exists and equals 1. } 6.5$$

(c) Show that $x^4 - 4x^3 + 6x^2 - 4x + 1$ has a maximum at $x = 1$. 6.5

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Your Roll No. :

Sl. No. of Q. Paper : **7099** **J**

Unique Paper Code : 62354343

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : Analytical Geometry and
Applied Algebra

Semester : III

Time : 3 Hours **Maximum Marks : 75**

Instructions for Candidates :

- (i) Write your Roll No. on the top immediately on receipt of this question paper.
- (ii) All questions are compulsory.
- (iii) Attempt any two parts from each question.

1. (a) Identify and sketch the graph

$$y = 4x^2 + 8x + 5$$

Also label the focus, vertex and directrix.
6.5

(b) Describe the graph of the curve

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.
6.5

P.T.O.



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- (c) Sketch the hyperbola

$$16x^2 - y^2 - 32x - 6y = 57$$

Also find its vertices, foci and asymptotes.

6.5

2. (a) Find an equation for the parabola that has vertex $(5, -3)$, axis parallel to the y -axis and passes through $(9, 5)$. 6

- (b) Find an equation for the ellipse that has ends of major axis $(\pm 6, 0)$ and passes through $(2, 3)$. 6

- (c) Find an equation for a hyperbola that satisfies the given conditions :

Asymptotes $y = 2x + 1$ and $y = -2x + 3$ passes through the origin. 6

3. (a) Consider the equation $xy = -9$. Rotate the coordinate axes to remove xy -term. Then identify and sketch the curve. 6.5

- (b) Let an $x'y'$ coordinate system be obtained by rotating an xy - coordinate system through an angle of $\theta = 60^\circ$.

- (i) Find the $x'y'$ - coordinates of the point whose xy - coordinates are $(-2, 6)$

- (ii) Find an equation of the curve

$$\sqrt{3}xy + y^2 = 6 \text{ in } x'y' \text{- coordinates.} \quad 6.5$$

- (c) (i) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 25 = 0.$$

(ii) Determine and sketch the surface represented by the equation $x^2 + y^2 = 25$ in 3-space. 6.5

4. (a) Find u and v if $5u + 2v = 6i - 5j + 4k$ and $3u - 4v = i + 2j + 9k$. Also find a vector of length 3 and oppositely directed to v . 6
- (b) (i) Show that direction cosines of a vector satisfy $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
- (ii) Determine if $u = < 6, 1, 3 >$ and $v = < 4, -6, -7 >$ make an acute angle, an obtuse angle or are orthogonal? Justify your answer. 6
- (c) Find the volume of the parallelepiped with adjacent edges $u = 3i + 2j + k$, $v = i + j + 2k$ and $w = i + 3j + 3k$. Also find the area of the face determined by u and v . 6

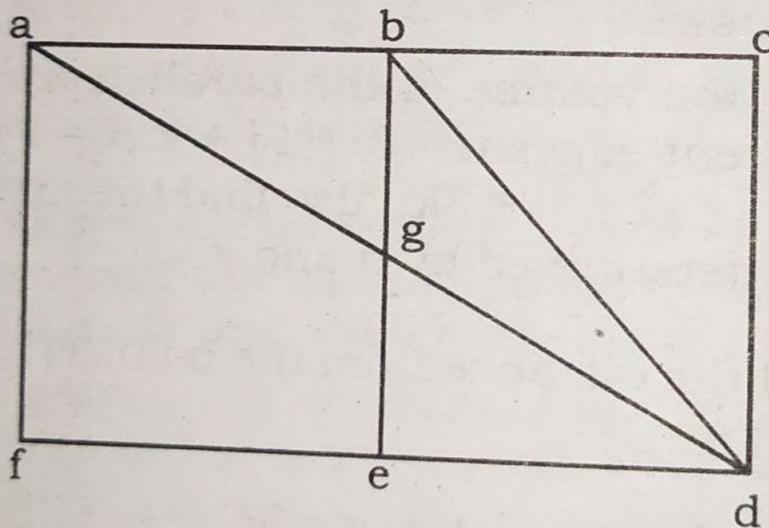
5. (a) Find the distance from the point $P(1, 4, -3)$ to the line 6.5

$$L : x = 2 + t, y = -1 - t, z = 3t.$$

- (b) Find the equation of the plane through the points $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$. 6.5

- (c) Find the distance between the skew lines
- $$L_1 : x = 1 + 4t, y = 5 - 4t, z = -1 + 5t;$$
- $$L_2 : x = 2 + 8t, y = 4 - 3t, z = 5 + t. \quad 6.5$$

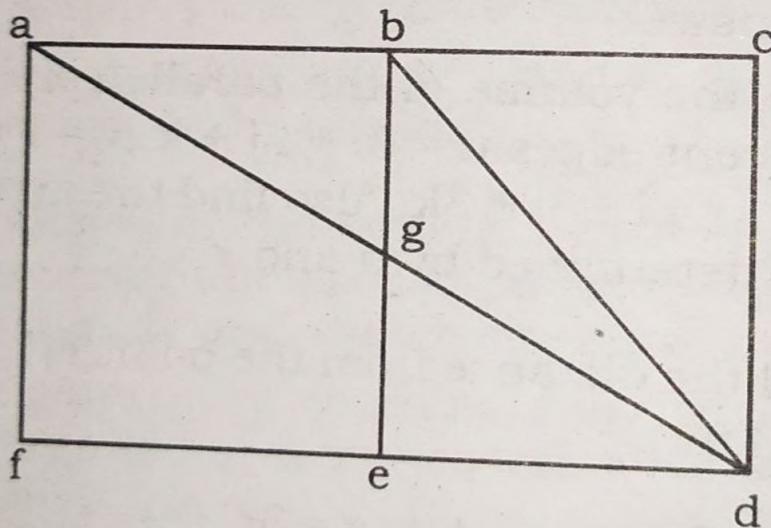
6. (a) Suppose there are three pitchers of water with capacity 8L, 5L and 3L. Initially, the 8 L pitcher is full and the other two are empty. Is there a way to pour among pitchers to obtain exactly 4 litres in 5L pitcher or 3L pitcher ? If so, find the minimal sequence of pourings to get 4 litres in either 5L pitcher or 3L pitcher. 6
- (b) For the following graph find a minimal edge cover and a maximal independent set of vertices 6



- (c) A supermarket wishes to test the effect of putting cereals on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of the cereal. 6



6. (a) Suppose there are three pitchers of water with capacity 8L, 5L and 3L. Initially, the 8 L pitcher is full and the other two are empty. Is there a way to pour among pitchers to obtain exactly 4 litres in 5L pitcher or 3L pitcher ? If so, find the minimal sequence of pourings to get 4 litres in either 5L pitcher or 3L pitcher. 6
- (b) For the following graph find a minimal edge cover and a maximal independent set of vertices 6



- (c) A supermarket wishes to test the effect of putting cereals on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of the cereal. 6



This question paper contains 4+2 printed pages]

Roll No.

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S. No. of Question Paper : 7229

Unique Paper Code : 62353505

J

Name of the Paper : Statistical Software R

Name of the Course : B.A. (Prog.) Mathematics : S.E.C

Semester : V

Duration : 2 Hours

Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

All commands should be written in software R.

1. Do any five of the following : $5 \times 1 = 5$

State whether the following statements are true or false :

(i) rm() command finds the defined variables.

(ii) colors() and colours() commands give the same output.

(iii) Quantile-Quantile plots are used for visualizing data in a straight line.

(iv) c (3 5 7 9) gives a vector.

P.T.O.

(2)

(v) sample() command selects random elements from

(vi) ls.str() command finds the structure of all the objects.

2. Do any five of the following :

Fill in the blanks :

(i) command is used to make scatter

(splot() / plot())

(ii) command can be used to view the type of an object. (summary() / class())

(iii) names() command is used for viewing.....
(rows/columns)

(iv) command is used to generate a sequence of 10 random numbers. (seq(10)/ rseq(10))

(v) Command for $\cot^{-1}(x)$ is..... (according to notes)
arccot(x))

(vi) command rearranges the items in a vector.

(3)

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3. Write the commands in R, for the following :

2x8=16

(a) (i) Read data from the file "hybrid.txt".

(ii) Using scan function, enter the following data :

Subject : Eng Sociology Science History

(b) (i) List the object starting with b or ending with t.

(ii) Save the commands in a file with name "commands."

Use data : 2 3 7 2 4 3 2 5 6 3 1 3 7 8; for question(c)

and (d)

(c) (i) Display the values less than 4 and greater than 6.

(ii) Count the items in the above sample.

(d) (i) Create a contingency table.

(ii) Create a stem and leaf plot.

Consider the following dataframe 'data', for questions (e)-(g) :

data	data 1	data 2	data 3
23	25	34	
43	32	56	
23	65	21	
34	76	78	
32	67	32	

(4)

- (e) (i) Print the first, third and fifth rows.
(ii) Sort the above sample.
(f) Convert the above data frame into a matrix with 'consumers'. Also, determine the structure of it.
(g) Add the row names : FY2012 FY2013 FY2014 FY2015 FY2016 to the above dataframe.

(h) Explain the command :

```
data1[seq(1, length(data1), 2)]
```

4. Do any four of the following :

(a) (i) Create the following data frame :

> bird

	Garden	Hedgerow
Blackbird	47	10
Chaffinch	19	3
Great Tit	50	0
Robin	9	3

(ii) Plot a bar chart of above data.

(iii) Alter the scale of the y-axis and add axis label from 1 to 100.

(5)

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(b) (i) Display the data frame :

> rainfall

	rain	Day
1	3	Mon
2	5	Tue
3	7	Wed
4	9	Thu
5	3	Fri

(ii) Plot above data, label the axes.

(iii) Enclose the whole plot in a bounding box.

(c) (i) Make a vector

data 1:3 5 7 6 5 7 3 3 8 4 2 7 1

(ii) Display three quantiles 20%, 50% and 80%.

(iii) Display three quantiles by suppressing the

headings.

(iv) What is the use of fivenum() command ?

(d) (i) Display the data frame :

> fw

	count	Speed
Taw	3	5
Torridge	5	3
Ouse	7	8
Exe	9	4
Pit	3	9

(ii) Display the mean values of each row and column in above sample data.

(iii) Explain the following:

> apply(fw, 1, mean, na.rm = TRUE).

(e) (i) Make a vector

Data 2 : 3 3 8 4 2 7 1 5 7 2 8 7.

(ii) Create a histogram for above data.

(iii) Specify the breaks of bars at nos. 2, 5, 6, 9.

(iv) Color the bars and suppress the main title for the histogram.

[This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : 7200 J

Unique Paper Code : 62353326

Name of the Course : B.A. (Prog.) Math : SEC

Name of the Paper : Mathematical
Typesetting System

Semester : III

Time : 2 Hours Maximum Marks : 38

Instructions for Candidates :

(a) Write your Roll No. on the top immediately
on receipt of this question paper.

(b) All questions are compulsory.

1. Fill in the blanks :

1×5=5

(i) The symbols may be used instead of a
pair of \$ signs.

(ii) The output of \$ \sqrt{3}{x+y} \$ is

(iii) The command \psset{unit=1.5} changes units
from

(iv) The commands is used to create a sector
of a circle.

P.T.O.



REDMI NOTE 7S
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(v) In beamer, the command is used to show the elements of a list one point at a time.

2. Answer any six parts from the following :

2.5×6=15

(i) Write the command in LaTeX to obtain the

$$\text{expression } \lim_{x \rightarrow \infty} \frac{\pi(x)}{x / \log x} = 1$$

(ii) Explain the command \psellipse(2,2)(1.5,1)

(iii) Write the output of the command :

$$\$ \$ \backslash \det V_n = \backslash prod_{1 \leq i < j \leq n} (x_i)$$

(iv) Typeset the following in LaTeX :

$$R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

(v) Write the output for the following command

(a) \ast, (b) \notin, (c) \Leftarrow,

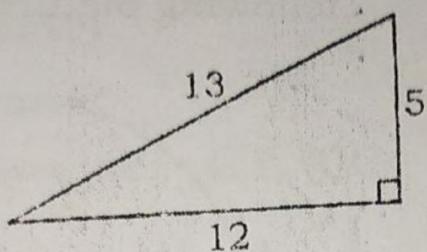
(d) \pm, (e) \cap .

(vi) Explain the command \pscircle(-2,1){2.5}.



REDMI NOTE 7S
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(vii) Write the command in PStricks to draw the following picture



(viii) Write the command in PStricks to plot the function $y = \cos(x)$.

3. Answer any four parts from the following :

$$4.5 \times 4 = 18$$

(i) Write the code to make the following multi-line equations

$$\begin{aligned} (a+b)^2 &= (a+b)(a+b) \\ &= (a+b)a + (a+b)b \\ &= a(a+b) + b(a+b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

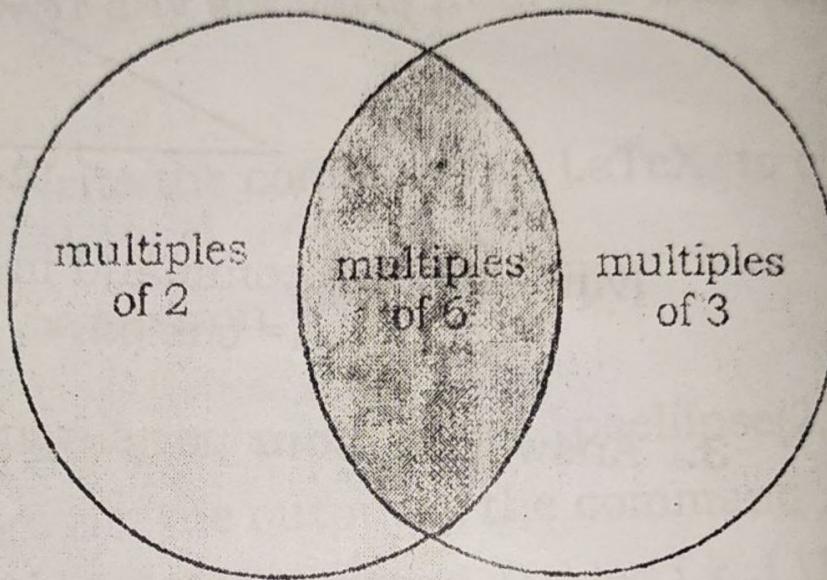
(ii) Write the code to typeset the following :

$$f(x) \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

P.T.O.

7200

- (iii) Write the code in PStricks to draw the following picture



- (iv) Write the code in PStricks to plot the cardioid given by the parametric equations :

$$x = \cos t (1 - \cos t)$$

$$y = \sin t (1 - \cos t), \quad 0 \leq t \leq 2\pi$$

- (v). Write a code to make a beamer presentation of 5 pages (including title and thank you page) on any topic with diagram/picture.

(75) This question paper contains 4 printed pages.

Your Roll No.

No. of Ques. Paper : 8227 J
Unique Paper Code : 32355101
Name of Paper : Calculus
Name of Course : Mathematics : G.E. (OC)
Semester : I
Duration : 3 hours
Maximum Marks : 75

(Write your Roll No. on the top immediately
on receipt of this question paper.)

All questions carry equal marks, 5 each.
Attempt any five questions from each Section.

SECTION I

Use $\epsilon-\delta$ definition to show that $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$.

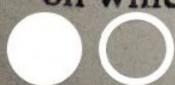
Find the horizontal and vertical asymptotes of the curve

$$f(x) = \frac{x-1}{x^2 + 2}$$

Find the linearization of $f(x) = \cos x$ at $x = -\frac{\pi}{2}$.

For $f(x) = 4x^3 - x^4$:

- (i) Find the intervals on which f is increasing and the intervals
on which f is decreasing.



- (ii) Find where the graph of f is concave up and where it is concave down.

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5. Use L'Hôpital's rule to find $\lim_{t \rightarrow 0} \frac{10(\sin t - t)}{t^3}$.

13

6. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

1

7. The radius r of a circle increases from $a = 10$ m to 10.1 m. Use dA to estimate the increase in the circle's area A . Estimate the area of the enlarged circle and compare your estimate to the true area.

SECTION II

8. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y -axis to generate a solid. Use washer method to find the volume of the solid.

9. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

10. Is the area under the curve $y = \frac{(\ln x)}{x^2}$ from $x = 1$ to $x = \infty$ finite? If so, what is it?

11. Use the comparison test to determine whether $\int_1^\infty \frac{dx}{\sqrt{x + e^{3x}}}$

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converges.

2. Sketch the graph of the curve $r = \cos 2\theta$ in polar coordinates.
3. If $\mathbf{r}(t)$ is a differentiable vector-valued function of t of constant length, then show that $\mathbf{r}(t)$ is orthogonal to $\frac{d\mathbf{r}(t)}{dt}$ for all t . Verify this result for the function $\mathbf{r}(t) = 3 \sin 5t\mathbf{i} + 9\mathbf{j} - 3 \cos 5t\mathbf{k}$.
4. Find the arc length parametrization for the helix $\mathbf{r}(t) = 4 \cos t\mathbf{i} + 4 \sin t\mathbf{j} + 3t\mathbf{k}$, $0 \leq t \leq \frac{\pi}{2}$.
- SECTION III
15. If $\mathbf{r}(t) = \left(\frac{t^3}{3}\right)\mathbf{i} + \left(\frac{t^2}{2}\right)\mathbf{j}$, $t > 0$, find binomial vector and torsion.
16. Find the limit of f as $(x, y) \rightarrow (0, 0)$ and show that limit does not exist for the function $f(x, y) = \frac{x-y}{x+y}$.
17. If $f(x, y) = \cos^2(3x - y^2)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
18. Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v if $z = 4e^x \ln y$, $x = \ln(u \cos v)$, $y = u \sin v$.
19. Find the directional derivative of the function f at P_0 in the direction of \mathbf{v} where $f(x, y, z) = \cos xy + e^{yz} + \ln zx$, $P_0\left(1, 0, \frac{1}{2}\right)$



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20. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point :

Surfaces : $x^2 + y^2 = 4$, $x^2 + y^2 - z = 0$, Point : $(\sqrt{2}, \sqrt{2}, 4)$.

21. A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of cross-section) does not exceed 108 inches. Find the dimensions of the acceptable box of largest volume.

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This question paper contains 4+1 printed pages]

Roll No.

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S. No. of Question Paper : 8352

Unique Paper Code : 32355301 J

Name of the Paper : Differential Equations

Name of the Course : Generic Elective : Mathematics

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt All questions by selecting any two parts from

each question.

1. (a) Show that the following first order ordinary differential equation :

$$(2x \cos y + 3x^2 y)dx + (x^3 - y - x^2 \sin y)dy = 0,$$

is exact and hence solve the equation with initial condition : $dy = 0$. 6.5

P.T.O.

- (b) By finding an integrating factor, solve the initial value problem : 6.5

$$(2x^2 + y)dx + (x^2y - x)dy = 0, \quad y(1) = 2.$$

- (c) Solve the following Bernoulli equation : 6.5

$$\frac{dy}{dx} + (x+1)y = e^{x^2}y^3, \quad y(0) = 0.5.$$

2. (a) Find the orthogonal trajectories of the family of parabolas $y^2 = 2cx + c^2$. Is the orthogonal trajectories also a family of parabolas ? 6

- (b) Solve the initial value problem :

$$y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -1. \quad 6$$

- (c) Find a basis of the following differential equation $(xD^2 + 4D)y = 0$, where $D \equiv d/dx$. Also find the solution satisfying :

$$y(1) = 12, \quad y'(1) = -6. \quad 6$$

3. (a) Solve by the method of variation of parameters :

$$y'' + 6y' + 9y = x^{-3}e^{-3x}, \quad x > 0.$$

6.5



- (b) Solve the initial-value problem by the method of undetermined coefficients : 6.5

$$y'' + 3y' + 2.25y = -10e^{-1.5x}, y(0) = 1, y'(0) = -1.$$

- (c) Find a homogeneous linear ordinary differential equation for which two functions $e^{-x} \cos x$ and $e^{-x} \sin x$ are solutions. Show also linear independence by considering their Wronskian. 6.5

4. (a) Solve the linear system that satisfies the stated initial conditions : 6

$$\frac{dy_1}{dt} = -3y_1 + 2y_2, \quad y_1(0) = 1$$

$$\frac{dy_2}{dt} = y_1 - 3y_2, \quad y_2(0) = -2.$$

- (b) (i) Find the partial differential equation arising from the surface : 3

$$z = xy + f(x^2 + y^2).$$

- (ii) Find the characteristics of the equation : 3

$$u_x - u_y = 1.$$

P.T.O.



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2020

- (c) Obtain the solution of the quasi-linear partial differential equation : 6

$$(y-u)u_x + (u-x)u_y = x - y,$$

with the condition $u = 0$ on $xy = 1$.

5. (a) Find a power series solution of the following differential equation : 6.5

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

- (b) Find the general solution of the linear partial differential equation : 6.5

$$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0.$$

- (c) Reduce the linear partial differential equation $u_x - yu_y - u = 1$ to canonical form, and obtain the general solution. 6.5

6. (a) Apply the method of separation of variables by taking $\log u(x,y) = f(x) + g(y)$, to solve the initial-value problem : 6

REDUCTION $y^2u_x^2 + z^2u_y^2 = (xyu)^2$, $u(x,0) = 3 \exp(x^2/4)$.



ANNUAL EXAMINATIONS

2020-10

- (b) Determine the region in which the partial differential equation :

$$u_{xx} + xyu_{yy} + u_x + u_y + u = 1,$$

is hyperbolic, parabolic or elliptic, and transform the equation into canonical form for the parabolic region.

6

- (c) Reduce the following partial differential equation with constant coefficients,

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$

into canonical form and hence find the general solution.

6

This question paper contains 4 printed pages]

414

Roll No.

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S. No. of Question Paper : 7264

Unique Paper Code : 62355503 J

Name of the Paper : General Mathematics—I

Name of the Course : Mathematics : Generic Elective

Semester : V

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt All questions as per directed question wise.

Section I

1. Write a short note on the life and contributions of any three
of the following mathematicians : 15

(a) Galois

(b) Reimann

(c) Poisson

(d) Cauchy

(e) Weierstrass.

P.T.O.

Section - II

2. Attempt any six questions. Each question carries five marks.

- (a) Define Magic Square and state the properties of Benjamin Franklin's Magic Square.
- (b) What is the total number of matches in a Round-Robin tennis tournament with 13 contestants ?
- (c) What is a Perfect number ? Give Euclid's Formula for finding a Perfect number. Is 28 a Perfect number ? Show that there is no odd Perfect number.
- (d) Define Unit Fraction and express $\frac{2}{5}$ and $\frac{98}{100}$ as sum of unit fractions.
- (e) State the Euclid's Algorithm. Using the above algorithm find the greatest common divisor of 6237 and 1549.
- (f) Show that the permutation (c e d b a) is an even permutation with the help of Inversions.
- (g) Define Algebraic and Transcendental numbers. Prove that transcendental numbers are irrational or both.



Section-III

3. Do any three questions. Each question carries six marks :

(a) If :

$$A = \begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix},$$

Show that $(AB)^2 \neq A^2 B^2$.

(b) Decompose the matrix :

$$\begin{pmatrix} 1 & 0 & -4 \\ 3 & 3 & -1 \\ 4 & -1 & 0 \end{pmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

(c) If $A = \begin{pmatrix} 3 & -1 \\ 4 & 7 \end{pmatrix}$, find A^4 .

(d) Find the adjoint and hence the inverse of the matrix :

$$\begin{pmatrix} -4 & 0 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

P.T.O.

4. Do any two questions. Each question carries six marks.

(a) If :

$$A = \begin{pmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{pmatrix},$$

find determinant of A.

(b) If $A = \begin{pmatrix} 5 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 3 \\ 1 & -15 \\ -2 & -1 \end{pmatrix}$

then $|AB| = |A| |B|$?

(c) Use Cramer's Rule to solve the following system of equations

$$3x - y - z = -8$$

$$2x - y - 2z = 3$$

$$-9x + y = 39.$$

