[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3199 **IC**

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : **B.A. (Program) Mathematics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Show that the set $W = \{(a_1, a_2, 0) : a_1, a_2 \in \mathbb{R}\}$ is a subspace of the vector space $\mathbb{R}^3(\mathbb{R})$.

(b) Show that the vectors $\{(1,1,-1), (2,-3,5), (-2,1,4)\}$ in $\mathbb{R}^3(\mathbb{R})$ are linearly independent.

(c) Let $\{a,b,c\}$ be a basis for $\mathbb{R}^3(\mathbb{R})$. Show that the set $\{a+b, b+c, c+a\}$ is also a basis of $\mathbb{R}^3(\mathbb{R})$.

$(2\times6=12)$

P.T.O.

2. (a) Using elementary transformations find rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}.$$

(b) Solve the following system of equations :

$$x - y + 2z = 4$$
$$3x + y + 4z = 6$$
$$x + y + z = 1.$$

(c) Find the characteristic roots and the characteristic vectors for the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$ (2×6½=13)

3. (a) Prove that $128\sin^2\theta\cos^6\theta = -\cos 8\theta - 4\cos 6\theta - 4\cos 4\theta + 4\cos 2\theta + 5$.

(b) Solve the equation $x^4 - 5x^3 + 10x^2 - 10x + 4 = 0$, given that the product of two of its roots is equal to the product of other two roots.

(c) Let α, β, γ be the roots of the equation $x^3 + qx - r = 0$. Find the values of :

$$\sum \frac{\beta+\gamma}{\alpha^2} \text{ and } \sum \alpha^2 .$$ (2×6=12)

4. (a) Solve the equation $z^4 - z^3 + z^2 - z + 1 = 0$ using De Moivre's theorem.

(b) If α, β, γ are the roots of $x^3 + px^2 + qx + p = 0$, prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$ radians, except in one particular case.

(c) Find the sum of the series

$$\sin\alpha + c\sin(\alpha+\beta) + \frac{c^2}{2!}\sin(\alpha+2\beta) + \ldots + \infty$$

(2×6½=13)

5. (a) Show that the set of positive rational numbers Q^+ forms an abelian group under the operation '*' defined by: a*b = ab/2. Also, write the inverse of ½ with respect to this operation.

(b) Draw the diagram associated with the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 8 & 1 & 6 & 3 & 7 & 2 & 4 & 9 \end{pmatrix}.$$

(c) Show that inverse of each element in a group is unique. (2×6=12)

6. (a) Let G be a group and H be a non-empty subset of G. Then show that the set H is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$.

(b) Show that under the usual operations of matrix addition and multiplication, the set

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\} \text{ is a ring.}$$

(c) With proper justification, give an example of a subring of $(\mathbb{Z}, +, \cdot)$. (2×6½=13)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **3234**	**IC**
Unique Paper Code	: **62354443**	
Name of the Paper	: **Analysis**	
Name of the Course	: **B.A. (Prog.) Mathematics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*All **six** questions are compulsory.*
*Attempt any **two** parts from each question.*

1. (a) Let A and B be two non empty bounded sets of positive real numbers and let

$$C = \{xy : x \in A \text{ and } y \in B\}.$$

Show that C is bounded and:

(i) $\text{Sup } C = \text{Sup } A \ \text{Sup } B$

(ii) $\text{Inf } C = \text{Inf } A \ \text{Inf } B$ 6

(b) If $y > 0$ is a real number, show that there exists a natural number n such that:

$$\frac{1}{2^n} < y.$$ 6

(c) Define limit point of a set $S \subseteq \mathbb{R}$. Find the limit points of the following sets:

P. T. O.

(i) $\mathbb{N}$

(ii) $\mathbb{R}$ 6

2. (a) If A and B are two open sets then prove that $A \cap B$ is also open. Is intersection of infinite sets of open sets again open? Justify. 6

(b) Test the continuity of function

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$. 6

(c) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2, 2]$. 6

3. (a) State Cauchy convergence criterion for sequences and hence show that the sequence $< x_n >$ defined by :

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots\dots + \frac{1}{2n-1}$$

does not converge. 6.5

(b) Show that $\lim_{n\to\infty}(r)^n = 0$ if $|r| < 1$. 6.5

(c) If $< x_n >$ and $< y_n >$ be two sequences such that $\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y$ then show that:

$$\lim_{n\to\infty}(x_n y_n) = xy.$$ 6.5

4. (a) State and prove limit comparison test for positive term series. 6.5

(b) Check the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\{(n^3+1)^{\frac{1}{3}}-n\}$

(ii) $\sum_{n=1}^{\infty}\frac{1.2.3.\ldots\ldots\ldots n}{7.10.13\ldots\ldots(3n+4)}$ 6.5

(c) State Leibnitz test for convergence of an alternating series:

$$\sum_{1}^{\infty}(-1)^{n-1}u_n,\ u_n>0\ \forall n$$

and check the convergence and absolute convergence of the series:

$$\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\cdots\cdots\cdots$$ 6.5

5. (a) Prove that a sequence cannot converge to more than one limit. 6

(b) Show that the sequence $<x_n>$ defined by:

$$x_1=1, x_{n+1}=\frac{3+2x_n}{2+x_n},\ n\geq 2$$

is convergent. Also find $\lim_{n\to\infty}x_n$. 6

(c) Show that:

$$\lim_{n\to\infty}\frac{1}{n}\left[1+2^{1/2}+3^{1/3}+\cdots\cdots+n^{1/n}\right]=1$$ 6

P. T. O.

6. (a) Apply Integral Test to examine the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ 6.5

(b) Prove that a continuous function is Riemann integrable. 6.5

(c) Discuss the integrability of the function f defined on $[0,1]$ as follows:

$$f(x) = \begin{cases} \frac{1}{2^n}, & \frac{1}{2^{n+1}} < x \le \frac{1}{2^n} \quad (n = 0,1,2,3,\ldots) \\ 0, & x = 0 \end{cases}$$

6.5

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **3376**	**IC**
Unique Paper Code	: **62357602**	
Name of the Paper	: **Numerical Analysis**	
Name of the Course	: **B.A. (Prog.) Maths.: DSE-1B**	
Semester	: **VI**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any ***two*** *parts from each question. Use of simple calculator is allowed.*

1. (a) Perform three iterations of Newton-Raphson method to obtain root of the equation:

$$f(x) = \cos x - xe^x = 0$$

with initial approximation $x_0 = 1$. 6

(b) Define Truncation Error. Evaluate the sum:

$$\sqrt{3} + \sqrt{5} + \sqrt{7}$$

to four significant digits and find its absolute and relative errors. 6

(c) Perform three iterations of Regula Falsi method to obtain the root of the equation:

P. T. O.

$$f(x) = x^3 - 2x^2 - 5 = 0(6)$$

in the interval $[2,3]$. 6

2. (a) Perform three iterations of secant method to obtain the square root of 3 with initial approximation:

$$x_0 = 1,\ x_1 = 2.$$ 6

(b) Perform four iterations of bisection method to find the root of the equation:

$$x^3 - 2x^2 - 0.04x + 0.08 = 0$$

in the interval $[0,1]$. 6

(c) Perform two iterations of Newton's method to solve the non-linear system of equations:

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

with initial approximation $(x_0, y_0) = (1.5, 0.5)$. 6

3. (a) Solve the following system of linear equations using the Gauss elimination method with partial pivoting:

$$2x + 6y + 10z = 0$$
$$x + 3y + 3z = 2$$
$$3x + 14y + 28z = -8$$

6.5

(b) Perform three iterations of Gauss-Seidel iteration method for the following system of equations:

$$5x + y - 2z = 2$$
$$3x + 4y - z = -2,$$
$$2x - 3y + 5z = 10,$$

starting with initial solution as $(x, y, z) = (0,0,0)$. 6.5

(c) Solve the following system of equations by using the Gauss-Jordan method:

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -3 & 5 \\ -3 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ -3 \end{pmatrix}$$ 6.5

4. (a) Prove the following relation:

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$$ 6

(b) Given the following data:

x	0.1	0.2
$\sin(x)$	0.09998	0.1986

Find Lagrange interpolating polynomial and approximate the value $\sin(0.15)$. Obtain a bound on the truncation error also. 6

(c) The following data represents the function $f(x) = x^{\frac{1}{3}}$:

x	0	1	8
$f(x)$	0	1	2

Find Newton divided difference interpolating polynomial of degree 2. Also find the approximate value of $f(7)$ and compare with the exact value. 6

5. (a) Find $f''(2.0)$ using quadratic interpolation using the following data:

x	2.0	2.2	2.6
$f(x)$	0.69315	0.78846	0.95551

Obtain an upper bound on error also. 6.5

(b) Find the approximate value of $I=\int_0^1 \frac{\sin x}{x}dx$. by using Newton's Cote open formula (i) mid- point rule (ii) two- point rule. 6.5

(c) Evaluate the integral $I=\int_0^2 \frac{dx}{3+4x}$, using Gauss Quadrature 3- point rule. 6.5

6. (a) Use the Euler method to solve boundary value problem

$$y'=4e^{0.8t}-0.5y,\ y(0)=2$$

on the interval $[0,3]$ with $h=1$. 6.5

(b) Given the initial value problem:

$$y'=-2ty^2\ ,\quad y(0)=1$$

estimate $y(0.4)$ using Ralston's method (R. K. Method 2nd Order) with $h=0.2$. 6.5

(c)Using a second order finite difference method with $h=1$, find the solution of the Boundary Value Problem $y''-y=x(x-4),\ 0\le x\le 4$

with $y(0)=y(4)=0$ .6.5

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This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : **3404** **IC**

Unique Paper Code : **62357602**

Name of the Paper : **Numerical Analysis**

Name of the Course : **B.A. (Prog.) Maths - DSE-2B**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question. Use of non-programmable simple calculator is allowed.

1. (a) Define Rounding-off error with examples. If 0.333 is the approximate value of $\frac{1}{3}$, find absolute and relative errors. 6

   (b) Perform two iterations of Newton's method to solve the non-linear system of equations:

$$y\cos(xy)+1=0$$
$$\sin(xy)+x-y=0$$

   with initial approximation $(x_0, y_0)=(1,2)$. 6

(c) Perform four iterations of bisection method to obtain the root of the equation:

$$x^3 - 3x^2 - 0.16x + 0.48 = 0$$

in the interval $[0,1]$. 6

2. (a) Perform three iterations of Regula Falsi method to find the root of the equation:

$$f(x) = x^2 - 5 = 0$$

in the interval $[2,3]$. 6

(b) Perform three iterations of Newton-Raphson method to find the square root of $3/4$ with initial approximation $x_0 = 0.5$. 6

(c) Perform three iterations of secant method to find the root of the equation:

$$x^3 - 9x + 2 = 0$$

in the interval $[0,1]$. 6

3. (a) Solve the following system of linear equations using the Gauss elimination method with partial pivoting:

$$x + 2y + 3z = 1$$

$$2x + 4y + 10z = -2$$

$$3x + 14y + 28z = -8.$$

6.5

(b) Perform three iterations of Gauss-Seidel iteration method for the following system of equations:

$$2x-y=7,\ -x+2y-z=1,\ -y+2z=1,$$

starting with initial solution as $(x, y, z) = (0,0,0)$. 6.5

(c) Explain Gauss-Thomas algorithm and solve the tri-diagonal system $AX = b$ by using Gauss-Thomas method.

$$\begin{pmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 41 \\ 25 \\ 105 \end{pmatrix}$$ 6.5

4. (a) Given that:

$$f(-2) = 4,\ f(0) = 2,\ f(2) = 8$$

Find the unique polynomial of degree 2 by Lagrange interpolation. Also find bound on the error. 6

(b) For the following data, calculate the differences and obtain the Gregory-Newton forward difference polynomial:

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Also find the approximate value of $f(0.35)$. 6

(c) (i) Find the maximum value of step size h that can be used to tabulate $f(x) = e^x$ on $[0,1]$ using linear interpolation such that $|error| \le 5 \times 10^{-4}$.

(ii) Let $f(x) = \log_e(1+x)$, $x_0 = 1$ and $x_1 = 1.1$.

Approximate $f(1.04)$ by using Newton divided difference interpolating polynomial. 6

5.(a) Find the approximate value of $I = \int_0^4 2^x dx$ using (i) Trapezoidal rule, (ii) Simpson's rule. 6.5

(b) Find the approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$ by using Newton Cote's open formula for three-point rule. 6.5

(c) Evaluate $\int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ using composite Trapezoidal rule and Romberg integration with $h = 1$ and $h = 1/2$ only. 6.5

Q6 (a) Apply Euler's modified method to approximate the solution of the following initial-value problem and calculate $y(2)$ by using $h = 1$:

$$y' = 4e^{0.8t} - 0.5y, \quad y(0) = 2$$ 6.5

(b) Employ the classical fourth order RK method to integrate $y' = -2ty^2$, $y(0) = 1$ with $h = 0.2$ on the interval $[0, 0.2]$. 6.5

(c) Apply finite difference method to solve the given problem:

$$\frac{d^2y}{dx^2} = y + x(x-4), \quad 0 \le x \le 4$$

with $h = 1$ and $y(0) = 0$, $y(4) = 0$. 6.5

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2301**

Unique Paper Code : **42353404** **IC**

Name of the Paper : **Computer Algebra Systems**

Name of the Course : **B.Sc. (Prog.)/B.Sc. Math. Sciences : SEC**

Semester : **IV**

Duration : **2 Hours** Maximum Marks : **38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Using any one of the CAS := Mathematica/Maple/Maxima/Matlab to answer the questions.

This question paper has *four* questions in all.

All questions are compulsory.

1. True/False (Give satisfactory Explanation/Example) : $8 \times 1 = 8$

 (*i*) Does the suffix ".nb" stand for "notebook" ?

 (*ii*) In Mathematica, every built-in function name begins with a small letter.

 (*iii*) Do the commands D[f[x]] and f'[x] provide the same output ?

P.T.O.

(iv) The syntax NSolve[$-1+3x+x^2=0$,x,15] is correct.

(v) The syntax Makelist[n^2,n,1,10,2]; is well defined.

(vi) The output of Factor [x^2-2] is $(x-\sqrt{2})(x+\sqrt{2})$.

(vii) x := RandomInteger[10]; {x,x,x} will give the same value of x in output.

(viii) Can we plot $y=4x+1$, $y=-x+4$ and $y=9x-8$, for $0\leq x\leq 2$ in a single graph ?

2. Attempt any *four* parts from the following : $4\times2\frac{1}{2}=10$

(i) What is the significance of simpsum command in the simplification to sums in maxima ?

(ii) Explain Reduce and Solve command.

(iii) Explain the use of 'Manipulate' command.

(iv) What is the use of command Direction → 1 in Limit command ? Can we change that value 1 with any other integer ?

(v) Define Matrix Form and Min command with suitable example.

(vi) Explain the role of Aspect Ratio and Plot Style of Plot options with syntax.

3. Write the Output of any *five* from the following : 5×2=10

(*i*) Plot [Sin[x],{x, 0, 2Pi}, Ticks → {{0, Pi, 2Pi}, {0, 0.5, 1}},

AxesLabel → {x, y}, PlotLabel → Sin[x]].

(*ii*) Solve :

([2*x+y–3*z=10,x+4*y+2*z=12,–x+y+z=0],[x,y,z]);

(*iiii*) $M = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, N = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 1 & 6 \end{bmatrix}$

M*N

(*iv*) A=DiagonalMatrix[{a,b,c},1}]// MatrixForm

B=Table[i+j, {i,4}, {j,4}] // MatrixForm

A+B

(*v*) $M = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$M_{[[2]]} = M_{[[2]]} + M_{[[1]]}$

M // MatrixForm

Transpose[M] // MatrixForm

(*vi*) Plot[{x,x^2},{x,0,4},PlotRange→{0,5},PlotStyle→

{Black,Directive[Thick,Dotted,Black]}]

(*vii*) f(x):= x+ sin(x);

'diff('f(x), x)=diff(f(x), x);

at ('diff ('f(x), x), x=0)= at (diff (f(x), x) , x=0);

(*viii*) wxplot2d(x^2 ,[x, 0, 4],[box,false]);

wxplot3d ([Cos(t), Sin(t), a], [t, 0, 2*%pi], [a, -1,1]);

4. Provide the Syntax of any *four* from the following : 4×2½=10

(*i*) Write the syntax for the plotting of unit sphere in any software.

(*ii*) Give the syntax for finding the Ist derivative and Indefinite integral of the function $f(x)=x^2+\cos x$ using any software.

(*iii*) Write the commands for the solution of the following equations without using solve command.

$x-2y=5$ and $4x-3y=4$.

(*iv*) Write the command for $\lim_{x\to 0} \frac{\cos x}{x}$ and $\lim_{x\to\infty} \frac{\sin x}{x}$.

(*v*) Provide the syntax of piecewise command with the help of example.

(*vi*) Write the syntax for the addition operation for any *two* matrices of 3*3 order in the form of matrices.

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2306**

Unique Paper Code : **42353604** **IC**

Name of the Paper : **Transportation and Network Flow Problems**

Name of the Course : **B.Sc. Programme/B.Sc. Math. Sciences : SEC**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **55**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *four* questions in all.

All questions are compulsory.

1. Hero Auto has three plants in *Gurugram, Haridwar*, and *Satyavedu*, and two major distribution centers in Delhi and Nagpur. The capacities of three plants during the next quarter are 500, 1000, and 700 cars and demands at the two distribution

centers are 1300 and 900 cars. The transportation costs per car on the different routes, rounded to the closest rupees, are given in the following table :

Table : Transportation Cost per Car

	Delhi (1)	Nagpur (2)
Gurugram (1)	Rs. 100	Rs. 235
Haridwar (2)	Rs. 120	Rs. 128
Satyavedu (3)	Rs. 122	Rs. 88

Formulate the Transportation Model. 5

2. Attempt any *five* parts from the following :

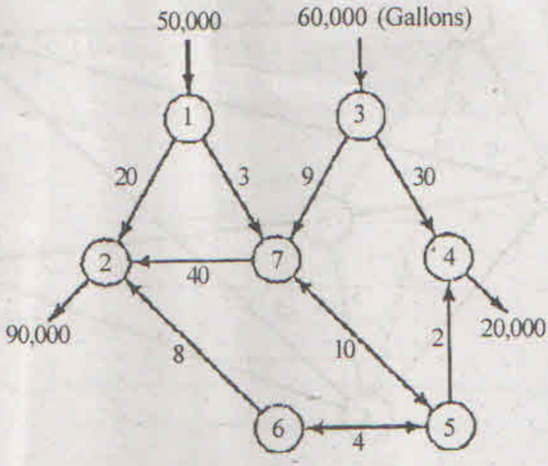
(*i*) Compare the initial basic feasible solutions obtained by the Northwest-Corner method and Least-Cost method for the following transportation problem : 3+3=6

	Destination				**Supply**
Source	11	13	17	14	**250**
	16	18	14	10	**300**
	21	24	13	10	**400**
Demand	**200**	**225**	**275**	**250**	

(*ii*) Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table. Find the assignment of men to jobs that will minimize the total time taken. 6

		Men				
		1	2	3	4	5
	A	3	8	2	10	3
	B	8	7	2	9	7
Jobs	C	6	4	2	7	5
	D	8	4	2	3	5
	E	9	10	6	9	10

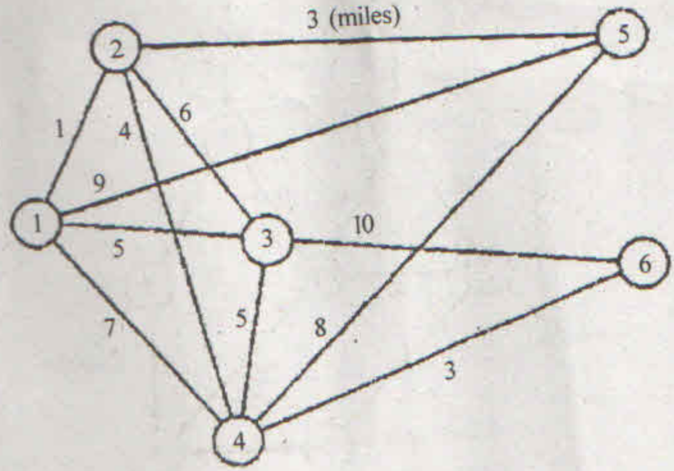
(*iii*) Consider the oil pipeline network shown in the following figure. The different nodes present pumping and receiving stations. Distances in miles between the stations are shown on the network :



(*a*) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(*b*) Only develop the corresponding transshipment model table. 2+4=6

(*iv*) Midwest TV Cable Company is in the process of providing cable services to five new housing development areas. The adjoining figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network starting at node 6. 6



(v) Draw the Network defined by the sets N=1,2,3,4,5,6} :

A={(1,2),(2,3),(3,4),(4,5), (5,6),(1,5),(1,3),(1,6),(3,6),(3,5)}

Also determine (a) a path (b) a cycle (c) a tree (d) a spanning tree. 6

(vi) The activities associated with a certain project are given below : 2+1+3=6

Activity	Predecessors	Duration (Week)
A	–	8
B	–	10
C	–	8
D	A	10
E	A	16
F	D, B	17
G	C	18
H	C	14
I	F, G	9

(a) Develop the associated network for the project.

(*b*) Find the minimum time of completion of the project.

(*c*) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table : 5+5=10

(*a*) Use the Vogel Approximation Method (VAM) to find a starting solution.

(*b*) Use this starting solution to find the optimal solution by the method of multipliers :

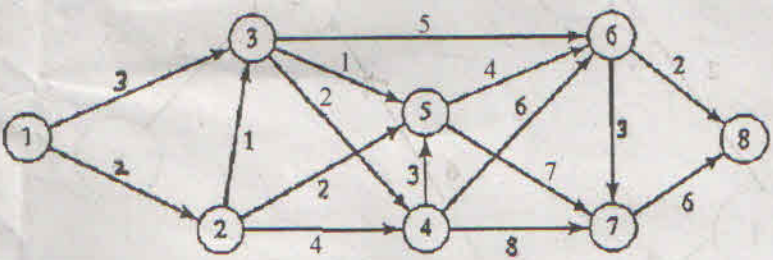
	X	Y	Z	Supply
A	5	1	8	12
B	2	4	0	14
C	3	6	7	4
Demand	9	10	11	

4. Attempt any *one* from the following :

(*i*) The network in the following figure gives the distances in miles between pairs of cities. Use Dijkstra's algorithm to find the shortest route between : 7+3=10

(*a*) cities 1 and 8

(*b*) cities 4 and 7.



(*ii*) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc(3, 4) is directional, so that no traffic is allowed from node 4 to node 3. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between :

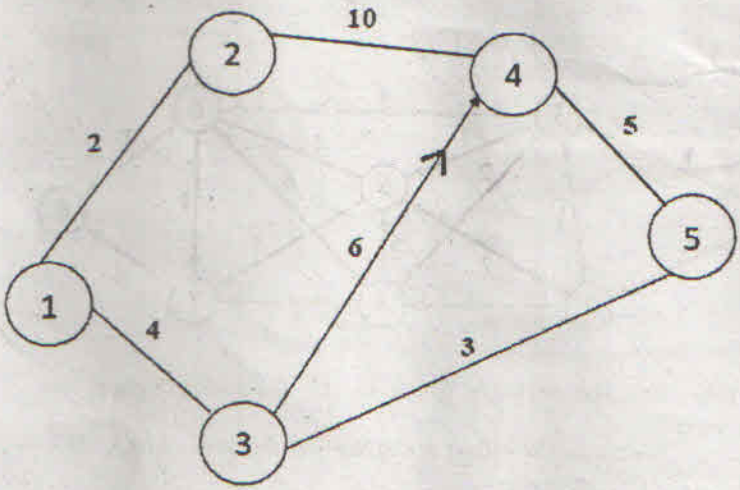
(*a*) node 5 to node 2

(*b*) node 1 to node 4

(*c*) node 2 to node 3

(*d*) node 3 to node 5

(*e*) node 1 to node 5 $5 \times 2 = 10$



[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3292 **IC**

Unique Paper Code : 62353606

Name of the Paper : Transportation and Network Flow Problem

Name of the Course : **B.A. Programe : Mathematics : SEC**

Semester : VI

Duration : 3 Hours Maximum Marks : 55

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **FOUR** questions in all.
3. **All** questions are compulsory.

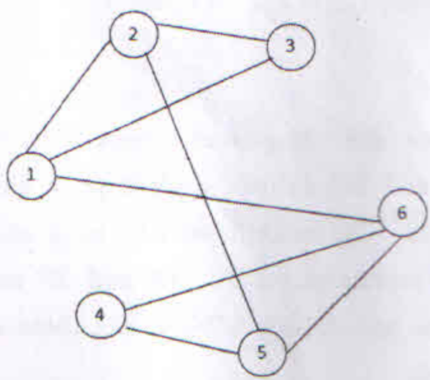
1. Three electric power plants with capacities of 20, 40 and 30 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 30, 35 and 25 million kWh. The price per million kWh at the three cities is given in Table

		City 1	City 2	City 3
Plant	1	\$600	\$700	\$400
	2	\$320	\$300	\$350
	3	\$500	\$480	\$450

The utility company wishes to determine the most economical plan for the distribution. Formulate the model as a transportation model. (5)

2. Attempt any **FIVE** parts from the following :

(i) For the Network given below, determine (a) a path (b) a cycle (c) a tree (d) a spanning tree (e) the sets N and A (6)



(ii) Compare the initial basic feasible solutions obtained by the Northwest-Corner method **AND** Least-Cost method for the following transportation problem. (3+3=6)

Warehouse

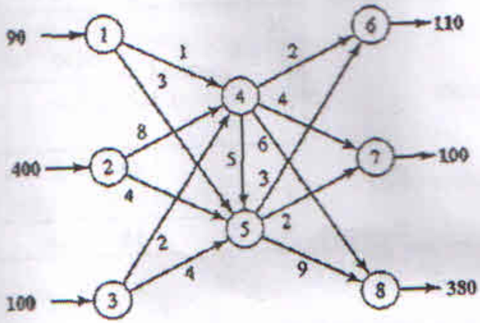
		1	2	3	4	Supply
Factory	1	10	2	20	11	15
	2	12	7	9	20	25
	3	4	14	16	18	10
	Demand	5	15	15	15	

(iii) Solve the following Assignment Problem using Hungarian Method. The Matrix entries represent the processing times in hours. (6)

Operators

		1	2	3	4	5
Jobs	1	9	11	14	11	7
	2	6	15	13	13	10
	3	12	13	6	8	8
	4	11	9	10	12	9
	5	7	12	14	10	14

(iv) The network in figure shows the routes for shipping cars from three plants (nodes 1, 2 and 3) to three dealers (nodes 6 to 8) by way of two distribution centers (nodes 4 and 5). The shipping costs per car (in \$100) are shown on the arcs.

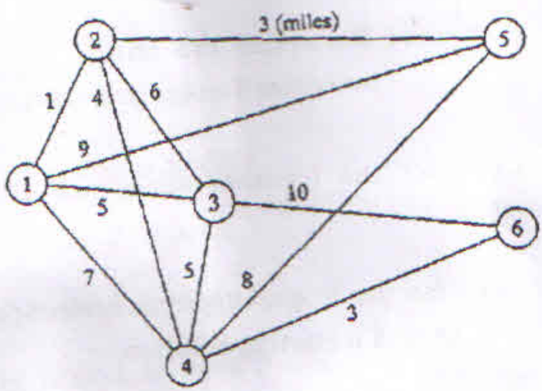


(a) Identify pure supply nodes, pure demand nodes, transshipment nodes and buffer amount.

(b) Only develop the corresponding transshipment model table. (2+4=6)

(v) A Company is in the process of providing cable service to five new housing development areas. (6)

Figure below depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network starting at node 4.



(vi) The activities associated with a certain project given below (2+1+3=6)

Activity	Predecessors	Duration (Week)
A:	--	4
B:	--	3
C:	A,B	2
D:	A,B	5
E:	B	6
F:	C	4
G:	D	3
H:	F,G	7
I:	F,G	4
J:	E,H	2

(a) Develop the associated network for the project.

(b) Find the minimum time of completion of the project.

(c) Determine the critical path and critical activities for the project network.

3. Consider the transportation model in the given table.

(a) Use the Vogel Approximation Method (VAM) to find a starting solution.

(b) Hence find the optimal solution by the method of the multipliers. (5+5=10)

	Destination			Supply
Source	\$0	\$2	\$1	6
	\$2	\$1	\$5	9
	\$2	\$4	\$3	5
Demand	5	5	10	

4. Attempt **ANY ONE** from the following :

(i) For the network given in the following figure, the distances (in miles) are given on the arcs. Arc (4, 5) is directional, so that no traffic is allowed from node 5 to node 4. List of all the other arcs allow two way traffic. Use Floyd's algorithm to determine the shortest route between (5×2=10)

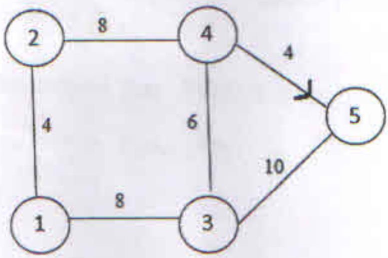
(a) node 5 to node 2

(b) node 1 to node 4

(c) node 2 to node 3

(d) node 3 to node 5

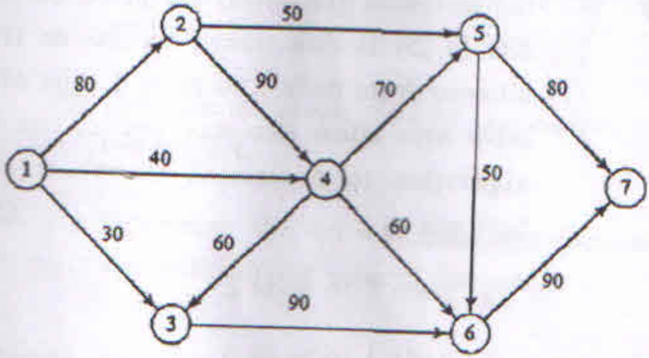
(e) node 1 to node 5



(ii) Use Dijkstra's algorithm to determine the shortest path (6+4=10)

(a) From node 1 to 5

(b) From node 2 to 7


2
5
7
1
4
3
6
50
80
90
70
80
40
50
30
60
60
90
90

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2980**

Unique Paper Code : **32355202** **IC**

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective—Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If x and y are vectors in $\mathbf{R}^3$, then prove that :

$$\|x\| - \|y\| \leq \|x + y\| \leq \|x\| + \|y\|.$$ 6½

(*b*) Let x and y be nonzero vectors in $\mathbf{R}^3$. If $x \cdot y \leq 0$, then prove that :

$$\|x - y\| > \|x\|.$$

Is the converse true ? Justify. 6½

P.T.O.

(*c*) Solve the following system of linear equations using the Gauss-Jordan method :

$$2x_1 + x_2 + 3x_3 = 16$$

$$2x_1 + 12x_3 - 5x_4 = 5$$

$$3x_1 + 2x_2 + x_4 = 16$$

6½

2. (*a*) Define the rank of a matrix and determine the rank of $\begin{pmatrix} 1 & 2 & -2 & -11 \\ 2 & 4 & -1 & -10 \\ 3 & 6 & -4 & -25 \end{pmatrix}$. 6

(*b*) Prove that the matrix $\begin{pmatrix} 7 & 1 & -1 \\ 11 & -3 & 2 \\ 18 & 2 & -4 \end{pmatrix}$ cannot be diagonalized. 6

(*c*) Let V be a vector space over **R**, then for any vector v in V and every nonzero real number a, prove that $av = 0$ if and only if $v = 0$. 6

3. (*a*) Let $S = \left\{ \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & -5 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} \right\}$ be a subset of 2×2 real matrices. Use the Simplified Span Method to find a simplified form for the vectors in span(S). Is the set S linearly independent ? Justify. 4½+2

(*b*) Define a basis for a vector space. Show that the set :

$$B = \{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$$

is a basis for $\mathbf{R}^3$. 2+4½

(*c*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

has a solution or not. If so, find the solution. 4+2½

4. (*a*) Consider the ordered basis S = {[1, 0, 1], [1, 1, 0], [0, 0, 1]} for $\mathbf{R}^3$. Find another ordered basis T for $\mathbf{R}^3$ such that the transition matrix from T to S is : 6

$$P_{S \leftarrow T} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}.$$

(*b*) Suppose $L : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator and $L([1, 1]) = [1, -3]$ and $L([-2, 3]) = [-4, 2]$. Express $L([1, 0])$ and $L([0, 1])$ as linear combinations of the vectors $[1, 0]$ and $[0, 1]$. 6

(*c*) Let $L : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation given by :

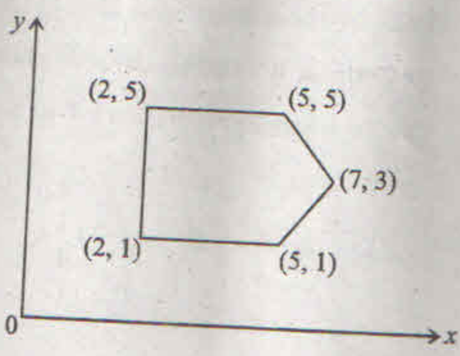
$$L([x, y, z]) = [-2x + 3z, x + 2y - z]$$

Find the matrix for L with respect to the bases :

$B = \{[1, -3, 2], [-4, 3, -3], [2, -3, 2]\}$ for $\mathbf{R}^3$

and $C = \{(-2, -1], [5, 3]\}$ for $\mathbf{R}^2$. 6

5. (*a*) For the graphic figure below, use homogeneous coordinates to find the new vertices after performing a scaling about the point (3, 3) with scale factors of 3 in the *x*-direction and 2 in the y-direction. Then sketch the final figure that would result from this movement : 4+2½



(b) Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{pmatrix} 3 & 1 & -3 \\ 2 & 1 & -1 \\ 2 & 3 & 5 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for ker(L) and a basis for range (L), also verify the dimension theorem. 4+2½

(c) Show that a mapping $L : P_2 \to P_2$ given by $L(p(x)) = p(x) + p'(x)$ is an isomorphism, where P_2 is the vector space of all polynomials of degree ≤ 2. 6½

6. (a) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $3x - y + 4z = 0$. Let $v = [2, 2, -3] \in \mathbf{R}^3$. Find $\text{proj}_{W^\perp} v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^\perp$. Is the decomposition unique ? 6

(*b*) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}$ of $\mathbf{R}^3$, find a basis for W and the orthogonal complement $W^{\perp}$. Also verify that :

$$\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3).$$ 4+2

(c) If $A = \begin{pmatrix} 1 & -1 \\ 4 & 1 \\ 2 & 3 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$, $z = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find vector v satisfying the inequality :

$$\| Av - b \| \leq \| Az - b \|.$$ 6

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **3104**

Unique Paper Code : **32355444** **IC**

Name of the Paper : **Elements of Analysis**

Name of the Course : **Mathematics : Generic Elective for Honours**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Define supremum and infimum of a bounded set of real numbers. Show that if S is a non-empty subset of **R** which is bounded below then : 7.5

$$\inf S = -\sup(-S).$$

P.T.O.

(*b*) Define a countable set. Show that S is countable if there exists a surjection of **N** to S. 7.5

(*c*) (*i*) State and prove Archimedean property of **R**.

(*ii*) Show that for $a, b \in \mathbf{R}$

$$\frac{|a+b|}{1+|a+b|} \leq \frac{|a|}{1+|a|} + \frac{|b|}{1+|b|}.$$ 3.5+4

2. (*a*) Let (x_n) be a sequence of positive real numbers such that :

$$l = \lim_{n \to \infty} \frac{x_{n+1}}{x_n}$$

exists. If $l < 1$, then show that (x_n) converges to 0. Hence deduce that :

$$\lim_{n \to \infty} \frac{n}{2^n} = 0.$$ 7.5

(*b*) State and prove squeeze theorem. Use it to determine the limit of the sequence $\left((n!)^{\frac{1}{n^2}}\right)$. 7.5

(c) (i) Define a convergent sequence and a Cauchy sequence. Show that every convergent sequence is a Cauchy sequence.

(ii) Show that the sequence (3^n) does not converge.

5+2.5

3. (a) State and prove Cauchy convergence criterion for infinite series. 6.5

(b) State root test for an infinite series and test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$

(ii) $\sum_{n=1}^{\infty} 2^n e^{-n}$. 6.5

(c) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n!}{3.5.7 \ldots\ldots (2n+1)}$

(ii) $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)}$. 3+3.5

P.T.O.

4\. (*a*) State limit comparison test for a positive term infinite series and hence test the convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$

(*ii*) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n+2}}$. 6

(*b*) Show that every absolutely convergent series in **R** is convergent. Is the converse true ? Justify. 6

(*c*) Test the convergence and absolute convergence of the following series :

(*i*) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^3}, \quad \alpha \in \mathbf{R}$

(*ii*) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+2}$. 3+3

5\. (*a*) Determine the interval of convergence for the power series :

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2}.$$ 5

(*b*) Derive the power series expansion for $f(x) = e^x$. 5

(*c*) Prove that :

$$\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2} \text{ for } |x| < 1$$

and evaluate :

(*i*) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(*ii*) $\sum_{n=1}^{\infty} \frac{n(-1)^n}{3^n}$. 5

6. (*a*) Find the radius of Convergence of the power series :

$$x - \frac{x^2}{2^2} + \frac{2!}{3^3}x^3 - \frac{3!}{4^4}x^4 + \ldots\ldots .$$ 5

(*b*) Find a power series representation of :

$$f(x) = \frac{1}{2+x}$$

and its domain. 5

(c) State differentiation theorem for power series. Show that :

$$C'(x) = -S(x) \text{ and } S'(x) = C(x), \quad x \in \mathbf{R}$$

where $C(x)$ and $S(x)$ are the power series of Sine and Cosine, respectively. 5

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3262 IC

Unique Paper Code : 62353424

Name of the Paper : Computer Algebra Systems

Name of the Course : **B.A. (Prog.) Mathematics : Skill Enhancement Course**

Semester : IV

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Using any one of the CAS :=Mathematica/Maple/ Maxima/any other to answer the questions.
3. This question paper has **four** questions in all.
4. **All** questions are compulsory.

1. Fill in the blanks : (8×1=8)

   (i) The ______ operator is to be used while assigning value 2 to a variable x.

   (ii) The output given on input π is ______ .

P.T.O.

(iii) The command which undoes the effect of Factor command is the ______ command.

(iv) The most recognized CAS ______ was created by Stephen Wolfram.

(v) The command ______ returns the nth derivative of f with respect to x.

(vi) The ______ brackets are used to group terms in algebraic expressions.

(vii) The command ______ is used to find the quotient when one polynomial is divided by another.

(viii) The option ______ causes the left hand limits to be computed by the 'Limit' command.

2. Write a short note on any **four** from the following : (4×2.5=10)

(i) How to find the limit of a function at a point in any CAS?

(ii) How to find maxima and minima of a function in any CAS?

(iii) How to differentiate a function in any CAS?

(iv) How to find eigenvalues and eigenvectors of a given 3×3 matrix in any CAS?

(v) Differentiate between the commands 'Solve' and 'NSolve'.

(vi) Differentiate between the commands 'AxesLabel' and 'PlotLabel'.

3. Write the output of any **five** from the following :

(i) Limit[$Sin[x], x \rightarrow$ Infinity]

Limit[$\frac{Sin[x]}{x}$, x → Infinity]

(ii) Solve [a x + b y = c,d x + e y = f, {x,y}]

Solve[x == 0]//Grid

(iii) x=RandomInteger[ ];

{2 x,2 x}

(iv) Plot[$x^{1/3}$,{x,-8,8}]

Plot[Cos[x],{x,0,Pi},Ticks → {Range[0,Pi,Pi/2],Automatic}]

(v) g[x_]: = x^3 – 9x + 5

Solve[g'[x] == 0,x]

extrema = {x,g[x]}/.%

(vi) 'diff(f(x) * g(x),x,2)=diff(f(x) * g(x),x,2);

'diff(diff(x^6,x),x)=diff(diff(x^6,x),x);

(vii) f[i_,j_]:=i^2+j^3;

(g=Array[f,{3,2}])//MatrixForm

(h=Array[Min,{3,2}])//MatrixForm

g+h//MatrixForm (5×2=10)

4. Provide the Syntax of any **four** from the following :

   (i) Write the manipulate command in the plotting of $f(x) = x^2 + \sin x$ using directive and blend commands.

   (ii) Write the command to sketch the graph of $f(x)=\frac{1}{x^2}$ and then evaluate the definite integral of f(x) from x=1 to x=3.

   (iii) Write the command to enter a matrix with the integers 1 through 5 on the diagonal, 0 below the diagonal, and 5 above the diagonal.

   (iv) Write the syntax for finding eigenvalues and eigenvectors of any 3×3 lower triangular matrix.

   (v) Write the command to get f′(0) and f″(1), where $f(x)=\frac{x^2}{1+x^3}$.

   (vi) Graph the functions $y = x\sin(1/x)$ and $z=\frac{xy}{x^2+y^2}$. (4×2.5=10)

This question paper contains 3 printed pages.

Your Roll No. **..............................**

S. No. of Paper	: **3343**	**IC**
Unique Paper Code	: **62355604**	
Name of the Paper	: **General Mathematics II**	
Name of the Course	: **B.A. (Prog.) Mathematics : GE**	
Semester	: **VI**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions as directed questionwise.*

SECTION I

1. Attempt any *five* questions out of the following:

   (a) Discuss the academic life of Hardy.

   (b) Write a short summary on Cantor's life.

   (c) What was Ramanujan's area of research?

   (d) When did Hilbert publish its first book? What was the foundation of that book?

   (e) Mention any *two* achievements of Banach.

   (f) Comment on the following statement: "Emmy Noether was never given her due academically because she was a woman, a Jew and a pacifist." 3×5

P. T. O.

SECTION II

2. Attempt any **six** questions out of the following:

(a) Define increasing and decreasing functions. Illustrate these functions with the help of examples.

(b) Define Circle and its diameter. What is the difference between tangent and secant of a circle? Also define segment and sector of the circle.

(c) Discuss all possible symmetries of an equilateral triangle.

(d) Show that the sequence of ratio of Fibonacci number to the one preceding it converges to the golden ratio.

(e) State the similarities and differences between Mobius strip and Klein bottle.

(f) Write short notes on any *two* of the following:

(i) Fractals

(ii) Chromatic Number

(iii) Konigsberg Bridge problem.

(g) If $\sin x = -5/13$, then find the other trigonometric angles.

5×6

SECTION III

3. Attempt any *five* questions from the following:

(a) Use Gauss Jordan method to convert the following matrix to reduced row echelon form:

$$\begin{bmatrix} 2 & 1 & 8 \\ 1 & -3 & 9 \\ 8 & 9 & 2 \end{bmatrix}.$$

(b) For the matrices $A = \begin{bmatrix} -4 & 10 & 0 \\ -3 & -5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -7 & 2 \\ 5 & -1 & 0 \end{bmatrix}$, find the rank of A+B.

(c) Solve the following homogeneous system of equations:

$$4x_1 - 8x_2 - 2x_3 = 0$$
$$3x_1 - 5x_2 - 2x_3 = 0$$
$$2x_1 - 8x_2 + x_3 = 0.$$

(d) Express the vector $x = [2,-1,4]$ as a linear combination of the other vectors, if possible:

$$a_1 = [3,6,2],\ a_2 = [2,10,-4].$$

(e) Find the inverse of the coefficient matrix and hence find the solution set of the system:

$$-7x_1 + 5x_2 + 3x_3 = 6$$
$$3x_1 - 2x_2 - 2x_3 = -3$$
$$3x_1 - 2x_2 - x_3 = 2.$$

(f) Use Gauss Elimination method to give the complete solution set of the following system of equations:

$$3x - 4y = 2$$
$$5x + 2y = 12$$
$$-x + 3y = 1.$$

(g) Find A^{-1} using row reduction for the matrix:

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 8 & -4 & 9 \\ -4 & 6 & -9 \end{bmatrix}.$$

5×6

200