[This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7948**

Unique Paper Code : **62351101** **HC**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1\. (*a*) Discuss the existence of the limit of the function : 6

$$f(x) = \frac{e^{\frac{1}{x^2}}}{1 - e^{\frac{1}{x^2}}}$$

at $x = 0$.

(*b*) Discuss the continuity of : 6

$$f(x) = |x - 1| + |x - 2|$$

at $x = 1$ and $x = 2$. Also state the kind of discontinuity, if any.

(*c*) Examine the following function for differentiability at $x = 0$: 6

$$f(x) = \begin{cases} x\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} ; & x \neq 0 \\ 0 \qquad ; & x = 0 \end{cases}$$

2. (*a*) Find the *n*th derivative of $\cos(x + 5)$. 6

(*b*) If 6

$$y = \left[x + \sqrt{1 + x^2}\right]^m,$$

prove that :

$$(1 + x^2)\, y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 - m^2)\, y_n = 0.$$

(c) If 6

$$u = \log\left(\frac{x^2 + y^2}{x + y}\right),$$

then using Euler's theorem, prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1.$$

3. (a) If the tangent to the curve : 6½

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$$

cuts off intercepts p and q from the axis of x and y respectively, show that :

$$\frac{p}{a} + \frac{q}{b} = 1.$$

(b) Find the equation of the tangent to the curve $y^2 = 4x$ which makes an angle 45° with the x-axis. 6½

(c) Show that radius of curvature is $4a\cos\frac{\theta}{2}$ for the cycloid : 6½

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta).$$

4. (*a*) Find the asymptotes of the curve : 6½

$$x^3 - 4x^2y + 5xy^2 - 2y^3 + 3x^2 - 4xy + 2y^2 - 3x + 2y - 1 = 0.$$

(*b*) Find the equation of the tangent to the curve : 6½

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$

at $(-1, -2)$, and show that it is a cusp.

(*c*) Trace the curve : 6½

$$x^3 + y^3 = 3axy, \ a > 0.$$

5. (*a*) State Lagrange's mean value theorem. Can we drop some condition of Lagrange's mean value theorem ? Justify your answer. 6

(*b*) Let $f(x) = \tan x$ for all x in $\mathbf{R}$. Using Lagrange's mean value theorem, for the function f, show that : 6

$$|\tan^{-1} x - \tan^{-1} y| < |x - y| \quad \forall \ x, y \in \mathbf{R}.$$

(c) Let f be a function defined by : 6

$$f(x) = x^3 - 6x^2 + 9x + 1 \quad \forall\, x \in \mathbf{R}.$$

Find the interval in which the function f is increasing or decreasing.

5. (a) Find the maximum and minimum values of the function : 6½

$$f(x) = 2x^3 - 15x^2 + 36x + 10 \quad \forall\, x \in \mathbf{R}.$$

(b) Define extremum of a function. Give an example of a function with no extremum. Justify your answer. 6½

(c) Evaluate : 6½

$$\lim_{x \to 0^+} (\cot x)^{\sin x}.$$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : 5190 **H**

Unique Paper Code : 235351

Name of the Course : **B.A. (Programme)**

Name of the Paper : Integration and Differential Equations

Semester : III

Time : Three Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each questions.

1. (a) Find the area of the region bounded above by y = x + 6 bounded below by $y = x^2$ and bounded on the sides by the lines x = 0 and x = 2. 6

(b) Evaluate : 6

(i) $\int \frac{2x+3}{\sqrt{3+4x-4x^2}}\,dx$

(ii) $\int \frac{dx}{4+5\sin x}$

P.T.O.

(c) Find the value of $\int_0^{\pi/4} \frac{\cos x - \sin x}{5+\sin 2x} dx$ 6

2. (a) Find the reduction formula for $I_{m,n} = \int \sin^m x \cos^n x \, dx$ where m and n are positive integers & hence evaluate

$\int \sin^4 x \cos^3 x dx$ $6\frac{1}{2}$

(b) Find the volume of the solid that results when the region enclosed by the given curve is revolved about the x-axis

$y = 9 - x^2$, $y = 0$. $6\frac{1}{2}$

(c) Find arc length of the curve $y = x^{2/3}$ from $x = 1$ to $x = 8$.

3. (a) Find the area of surface generated by revolving the given curve about the x-axis $y = \sqrt{4-x^2}$ 6

(b) Solve :

$ydx - xdy + \log x \, dx = 0$

(c) Find the orthogonal trajectories of the family of curves $y = cx^2$ where c is a parameter. 6

4. (a) Solve :

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right) \qquad 6\frac{1}{2}$$

(b) Solve :

$$\frac{d^2y}{dx^2} - \frac{6}{x^2}y^2 = x\log x \qquad 6\frac{1}{2}$$

(c) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. What is the general solution ? Find the solution y(x) satisfying y(0) = 2, y(0) = -3. $6\frac{1}{2}$

5. (a) A certain culture of bacteria grows at a rate that is proportional to the number present. It is found that the number doubles in 4 hours. How many may be expected at the end of 12 hours ? 6

(b) Solve 6

(yz + 2x) dx + (zx + 2y) dy + (xy + 2z) dz = 0

(c) Solve the following differential equation by method of variation of parameter

$$\frac{d^2y}{dx^2} + y = \tan x. \qquad 6$$

6. (a) Form the partial differential equation by eliminating the constants a, b from the equation.

(i) $z = ax + by + a^4 + b^4$

(ii) $z = (x + a)(y + b)$

$6\frac{1}{2}$

(b) Find the general solution of the Lagrange's equation $6\frac{1}{2}$

$x(y - z)p + y(z - x)q = z(x - y)$

(c) Solve the partial differential equation by Charpit's method

$(p^2 + q^2)y = qz$

$6\frac{1}{2}$

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **5191** **H**

Unique Paper Code : 235351

Name of the Course : **B.A. (Programme)**

Name of the Paper : Integration and Differential Equations

Semester : III

Time : Three Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **any two** parts from each questions.

1. (a) Find the area of the region enclosed by curve $y^2 = 4x$ and $y = 2x - 4$ by integrating with respect to x. 6

(b) Evaluate 6

$$\int \frac{dx}{\sqrt{(x^2+2x+5)}}$$

$$\int \frac{dx}{5+4\cos x}$$

P.T.O.

(c) Find the value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ 6

2. (a) Find the reduction formula $I_{m,n} = \int \cos^m x \sin nx \, dx$ where m and n are positive integers. $6\frac{1}{2}$

(b) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x -axis. $6\frac{1}{2}$

(c) Find the arc length of the curve $x = \frac{1}{3}\left(y^2+2\right)^{3/2}$ from y = 0 to y = 1. $6\frac{1}{2}$

3. (a) Find the area of surface generated by revolving the given curve about x-axis

$y = \sqrt{x} \qquad 1 \le x \le 4$ 6

(b) Solve 6

$$y\left(x^2y^2+2\right)dx + x\left(2-2x^2y^2\right)dy = 0$$

(c) Solve 6

$$y = 2px + y^2p^3, p = \frac{dy}{dx}$$

4. (a) Solve $6\frac{1}{2}$

$$\left(e^{x}+1\right)y\ dy=\left(y+1\right)e^{x}dx$$

(b) Find orthogonal trajectories of the family of curves $cx^2 + y^2 = 1$ where c is a parameter.

$6\frac{1}{2}$

(c) Evaluate Wronskian of the functions $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ and hence concluded whether or not they are linearly independent. Also form the differential equation.

$6\frac{1}{2}$

5. (a) A bacteria culture is known to grow at a rate proportional to the amount present. If the initial number is 300 and if it is observed that the population has increased by 20 percent after 12 hours determine the number of bacteria present in the culture after 2 days. 6

P.T.O.

(b) Solve the system of equations 6

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dx}{dt} + 2y + y = 3e^t$$

(c) Solve the differential equation by the method of variation of parameter :

$$\frac{d^2y}{dx^2} + y = \tan x$$ 6

6. (a) (i) Form a partial differential equations by eliminating the functions from $z = (x + y) + f(xy)$.

(ii) Eliminate the constants from $z^2 = ax^2 + by^2 + 1$ to form a partial differential equation. $6\frac{1}{2}$

(b) Find the general solution of following

$$y^2 - x^2 = z\,(xp - yq)$$ $6\frac{1}{2}$

(c) Solve the equations by Charpit's method

$$(p^2 + q^2)\,y = qz$$ $6\frac{1}{2}$

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7987**

Unique Paper Code : **62354343** **HC**

Name of the Paper : **Analytical Geometry and Applied Algebra**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Identify and sketch the curve :

$$(x + 2)^2 = -(y + 2)$$

and also label the focus, vertex and directrix. 6

(*b*) Sketch the ellipse :

$$9(x - 3)^2 + 25(y + 1)^2 = 225$$

also label foci, vertices and ends of major and minor axes. 6

(*c*) Describe the graph of the equation :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$ 6

2. (a) Find the equation of the parabola that has its vertex at (1, 1) and directrix $y = -2$. Also state the reflection property of parabola. 6

(b) Find an equation for the ellipse with length of major axis 10 and with vertices (3, 2) and (3, – 4) and also sketch it. 6

(c) Find and sketch the curve of the hyperbola whose asymptotes are $y = 2x + 1$ and $y = -2x + 3$ and the hyperbola passes through the origin. 6

3. (a) Consider the equation $x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$. Rotate the coordinate axes to remove the xy term and then identify the type of the conic represented by the above equation. 6

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 60°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinates are (2,6).

(*ii*) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinates. 6

(*c*) Find the equation of the sphere with center at (2,–1,–3) and is tangent to the zx-plane. 6

4. (*a*) (*i*) Find a vector **v** having opposite direction as the vector from the point P (1, 0, –6) to Q (–3, 1, 1) with $\|\mathbf{v}\| = 5$.

(*ii*) Sketch the surface $z^2 + y^2 = 4$ in 3-space. 3+3½

(*b*) (*i*) Using vector, find the area of triangle with vertices A(2, 2, 0), B(–1, 0, 2) and C(0, 4, 3).

(*ii*) Let $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find the volume of the parallelopiped with adjacent edges **u**, **v** and **w**. 3+3½

(*c*) Prove that

$$\mathbf{u}.\mathbf{v} = \tfrac{1}{4}\left(\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2\right).$$ 6½

5. (*a*) Find the distance between the skew lines : 6½

$$L_1 : x = 1 + 7t \quad y = 3 + t \quad z = 5 - 3t, \quad -\infty < t < \infty$$

$$L_2 : x = 4 - t \quad y = 6 \quad z = 7 + 2t, \quad -\infty < t < \infty$$

(b) (i) Determine whether the points P_1 $(-6, 4, 8)$, $P_2(9, -2, 0)$ and P_3 $(1, -5, 3)$ lie on the same line.

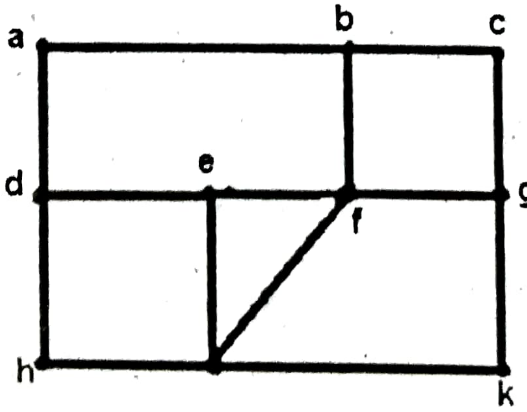
(ii) Where does the line

$$x = 2 - t,\ y = 3t,\ z = 1 + 2t$$

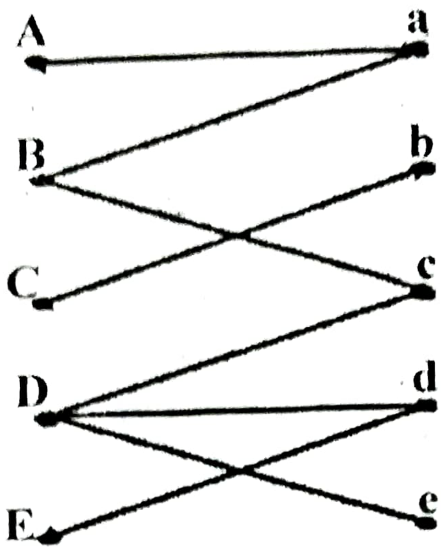
intersect the plane $2x - 7y + 3z = 6$. 3+3½

(c) Find the equation of the plane through the points $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$. 6½

6 (a) Find a maximum independent set of vertices for the following graph. What is the minimum number of independent set needed to cover all the vertices ? 6½



(*b*) (*i*) Find a matching or explain why none exists for the following graph :



(*ii*) Given three pitchers: 8, 5 and 3 liters capacity. Only 8 liter pitcher is full. Make at least one of them contain exactly 4 liter of water with the minimum number of water transfers. 3+3½

(*c*) Defing Latin square. Construct a Latin square of order 5 on $\{e, e^2, e^3, e^4, e^5\}$. 6½

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : **5230** **H**

Unique Paper Code : **235551**

Name of Paper : **Analysis**

Name of Course : **B.A. Programme**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

*There are three Sections. Each Section consists of 25 marks. Attempt any **two** parts from each question in each Section. Marks are indicated againt each question.*

SECTION I

1. (a) Define a bounded set, its supremum and infimum. Find the supremum and infimum of the following sets:

(i) $\left\{\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3} \text{.........}\right\}$

(ii) $\left\{\frac{n}{n+1}; n=1, 2, 3, \text{.........}\right\}$

(iii) Z, the set of integers. 6

(b) Define open set and prove that the union of an arbitrary family of open sets is an open set. 6

(c) Give an example of a set which has:

(i) No limit point

(ii) Unique limit point

(iii) Infinite number of limit points. 6

2. (a) Show that the function f defined as:

$$f(x)=\begin{cases}(1+2x)^{\frac{1}{x}}, & \text{when } x\neq 0\\ e^2, & \text{when } x=0\end{cases}$$

is continuous at $x = 0$. 6½

(b) Show that $f(x)=\frac{1}{x}$ is not uniformly continuous on [0, 1]. 6½

(c) (i) Define neighbourhood.

(ii) Define closed set.

(iii) Give an example of a set whose derived set is void. 6½

SECTION II

3. (a) Show that $\lim_{n\to\infty} r^n = 0, \text{if } |r|<1.$ 6½

(b) If $\langle a_n\rangle$ and $\langle b_n\rangle$ be two sequences such that:

$$\lim_{n\to\infty} a_n = a,\ \lim_{n\to\infty} b_n = b,\ b_n \neq 0 \text{ and } b\neq 0$$

then show that:

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)=\frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}=\frac{a}{b}.$$ 6½

(c) Prove that a monotone sequence is convergent iff it is bounded. 6½

(a) If $\sum_{1}^{\infty} u_n$ is a convergent series then show that $\lim_{n\to\infty} u_n = 0$. Does the converse of this result hold? Justify your answer. 6

(b) State Raabe's test for convergence of the series $\sum_{1}^{\infty} u_n$ and hence test the convergence of the series:

$$\sum_{1}^{\infty} \frac{1.3.5 \ldots\ldots\ldots (2n-1)}{2.4.6 \ldots\ldots\ldots 2n} \cdot \frac{1}{n}.$$ 6

(c) Test the absolute convergence of the following series:

(i) $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$

(ii) $\sum_{1}^{\infty} \frac{\sin nx + \cos nx}{n^{3/2}}$

(iii) $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n}$. 6

Section III

(a) Show that continuous function f defined on a closed and bounded interval $[a, b]$ is integrable. 6

(b) Test the convergence of the improper integral:

$$\int_0^{\infty} x^{n-1} e^{-x}\, dx.$$ 6

(c) Define Gamma function and show that $\int_{-\infty}^{\infty} e^{-x^2}$

6. (a) Find the Fourier series of the function f defi[ned as] follows:

$$f(x)=\begin{cases}1, & \text{for } -\pi < x \leq 0 \\ -2, & \text{for } 0 < x \leq \pi\end{cases}.$$

(b) Show that $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n^p\left(1+x^{2n}\right)}$ converges absolutel[y and] uniformly for all real values of x if $p > 1$.

(c) (i) Find the radius of convergence of the power s[eries]

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} x^n.$$

(ii) Discuss the Riemann integrability of the fu[nction] $f(x) = |x|$ on $[-1, 1]$.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8078**

Unique Paper Code : **62357502**

HC

Name of the Paper : **Differential Equations**

Name of the Course : **B.A. (Prog.) DSE : Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$(ye^x + 2e^x + y^2)\, dx + (e^x + 2xy)\, dy = 0;\ y(0) = 6.$$

(*b*) Solve : $(x^2 + y^2 + x)dx + xydy = 0$. 6

(*c*) Solve : $(x - 2y + 5)dx - (2x + y - 1)dy = 0$. 6

2. (*a*) Solve : $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = e^{2x} + x^2 + x$ 6.5

(*b*) Solve : $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2 \sin \ln x$. 6.5

(*c*) Consider the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0.$$ 6.5

(*i*) Show that each of the functions e^x, e^{4x} and $2e^x - 3e^{4x}$ is a solution. Also show that e^x and $2e^x - 3e^{4x}$ are linearly independent.

(*ii*) Write the general solution.

3. (*a*) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} + y = \sec^2 x.$$ 6.5

(*b*) Using the method of undetermined coefficients to find the general solution of the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}.$$ 6.5

(*c*) Given that $y = e^x$ is a solution of the differential equation :

$$x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$ 6.5

Find a linearly independent solution by reducing the order and write the general solution.

4. (*a*) Solve : $\dfrac{dx}{x(y^2 - z^2)} = \dfrac{dy}{y(z^2 - x^2)} = \dfrac{dz}{z(x^2 - y^2)}$. 6

(b) Solve : $yz(y + z)dx + xz(x + z)dy + xy(x + y)dz = 0.$ 6

(c) Solve : 6

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dx}{dt} + 2x + 5y = e^t.$$

5. (a) Find the general solution of the differential equation

$$(y + z)p + (z + x)q = x + y.$$ 6.5

(b) Find the complete integral of the differential equation

$$(p^2 + q^2)x = pz.$$ 6.5

(c) (i) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$u_{xx} + (1 + x^2)^2 u_{yy} = x^2.$$ 2.5

(ii) Eliminate the parameters a and b from the following equation to find the corresponding partial differential equation :

$$ax^2 + by^2 + z^2 = 1.$$ 4

6. (*a*) Find the complete integral of the equation :

$$p^2z^2 + q^2 = 1.$$ 6

(*b*) Eliminate the arbitrary function f from the equation $z = f\left(\frac{x}{y}\right)$ to find the corresponding partial differential equation. 6

(*c*) Find the general solution of the partial differential equation :

$$yzp + xzq = x + y.$$ 6

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **6820A** **HC**

Unique Paper Code : 42353327

Name of the Course : **Mathematics Skill Enhancement Course**

Name of the Paper : Mathematical Typesetting System : LaTeX

Semester : III

Time : 2 Hours ***Maximum Marks : 38***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

1. Fill in the blanks any **four** parts from the following : $4 \times 0.5 = 2$

(i) The command .... in the LaTeX document produces a medium space.

(ii) The .... command tells TeX to print its entire argument on the same line.

(iii) Indentation can be prevented in a paragraph of a LaTeX document with the .... command.

P.T.O.

(iv) In pspicture environment, the command .... produces an ellipse centered at (0,0) with major axis 6 units and minor axis 4 units.

(v) A mathematical formula appearing in the display mode is enclosed by .... and .... commands.

2. Answer any **eight** parts from the following :

$8 \times 2 = 16$

(i) Write the difference between \hspace and \hspace* commands.

(ii) Typeset the following in a displayed formula :

$$\underbrace{a+\overbrace{b+\ldots+y}^{24}+z}_{26}.$$

(iii) Explain the \qbezier command in the LaTeX picture environment.

(iv) Draw a square of side 4 units with reference point(1,–2) and rounded corners.

(v) Write the command to draw an arrow at (4,4) of length 10 units in the direction of positive x-axis.

(vi) In PS Tricks picture environment, write a command to change unit-length of x-axis and y-axis by 2 centimeter and 3 centimeter, respectively.

(vii) Give the command in LaTeX to produce an expression :

$$\frac{1}{b-a}\int_a^b f'(x)\,dx = \frac{f(b)-f(a)}{b-a}.$$

(viii) Write the code in LaTeX in display math mode to produce an output.

If $x \not> y$ then $x \ngeq y+1$.

(ix) Write the following postfix expression in standard from :

x sin 1 x cos 2 exp add div 3 exp.

(x) Give a command to draw sector of a circle of radius 2 units centered at (3,3), going from reference angle 0 to 60 degrees.

3. Answer any **three** parts from the following : 4+4+4=12

(a) Plot step function $f(x) = [x]$, $0 \leq x < 5$ in the picture environment.

(b) Write the code in LaTeX to obtain an expression :

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

$$e^x = \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \ldots\ldots$$

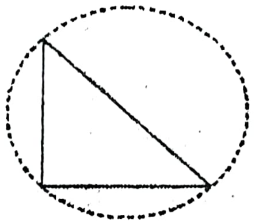
$$e^{-1} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots$$

P.T.O.

(c) Make the following equation in LaTeX u
delimiters :

$$\begin{vmatrix} \hat{i} & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

(d) Write a code in LaTeX using PSTric
draw the following :



4. Write a presentation containing in beamer
the following content.

Slide-1 : Title of the presentation with au
and date.

Slide-2: Fermat's Last Theorem. Let n>
any interger, then the equation
$x^n + y^n = z^n$
has no solutions in positive inte
for any x, y and z .

Slide-3 : This result is called his last theor
because it was the last of his cla
in the margins to be either prove
disproved. Andre Wiles found the
accepted proof in 1995, some
years later, Wiles proof
exceptionally long and difficult !

Slide-4: Thank you

is question paper contains 6 printed pages.

Your Roll No. ....

No. of Ques. Paper : 6825 A HC

ique Paper Code : 42353503

me of Paper : Statistical Software R

me of Course : Mathematics : Skill Enhancement Course

mester : V

ration : 2 hours

ximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions.

All commands should be written in software R.

Do any *four* of the following: 1x4

State whether the following statements are true or false:

(a) savehistory(file=' .Rhistory') is same as history( ) command.

(b) ls.str( ) is used to find the structure of all the defined objects.

(c) c(3 5 7 9) gives a vector.

(d) The commands mean( ) and colMeans( ) for a data frame give the same output.

(e) Pie chart cannot be formed of the data given in matrix form.

2. Do any *six* of the following. 1x6

Fill in the blanks:

(a) .... command to find the variance of data.

(var( )/ vara( ))

(b) .... command is used to make scatter plot.

(splot() / plot())

(c) $ command is used for ....

(copy a data, extract from a data).

(d) hist() command is used for .... (history, histogram).

(e) sample() command selects .... elements from data.

(random, beginning).

(f) rep() command is used for repeat .... items.

(one, multiple).

(g) .... command to rearrange the items in a vector to be in a order.

(sort, order)

3. Answer the following questions: 2x8=16

(a) (i) Write a command to list all the variables defined ending with 'm'.

(ii) Write "Jan", "Feb", "Mar", "Apr", "May" as a factor.

(b) (i) Can we use scan( ) command for the text Ajay, Anil, Raju, Ravi, Sanjay? Justify your answer.

(ii) What are the differences between save( ) and load( ) commands for files?

(c) Differentiate between seq(5) and seq_along(5) commands.

(d) Create a pie chart of any data with labels with one example.

(e) Rearrange the data in increasing order and draw a stem and leaf plot, where data is:

X = 3,5,7,5,3,2,6,8,5,6,9

(f) A data file is given with name bird.

	A	B	C	D	E
X	12	14	15	40	10
Y	08	04	07	09	11
Z	30	20	25	10	35

(i) Extract third columns.

(ii) Transpose bird data.

(iii) Find max and min items.

(iv) Make histogram of X.

(g) Make a score data file:

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Draw a stem leaf plot.

(h) Consider the data2 = 6,7,3,5,6,6,7,9,3,6. Write the sequence of first four positioned items of data2 and also write a sequence having the last four positioned items of data2.

Do any *four* of the following: 3×4

(a) Write commands for the following statements:

(i) Create a sequence of 25 numbers which are incremented by 1.

(ii) Create the binomial distribution of 25 numbers with probability 0.5.

(iii) Find the probability of getting 15 or less heads from a toss of a coin. (using binomial distribution)

(iv) How many heads will have a probability of 0.2 will come out when a coin is tossed 51 times.

(v) Find 8 random values from a sample of 150 with probability of 0.4. (using binomial distribution).

(b) Consider the following data frame object "x":

	C1	C2
R1	4	7
R2	3	4
R3	2	3
R4	4	4
R4	4	2
R5	6	3

Write commands for the following statements:

(i) Find the minimum and maximum value of the data frame x.

(ii) Find the column means and column sums of x.

(iii) Find the row mean of x.

(iv) Create a scatter chart of x .

(v) Create a line chart plot of vector C1.

(c) Make a dataframe file:

81	81	96
95	98	73
92	79	82
80	86	89
79	62	NA

Then convert this into a matrix

(d) Generate 50 random variables using normal distribution, negative-binomial distribution.

(e) Consider the following course grades of randomly selected students:

32	40	20	31
26	35	38	21
12	44	22	45
42	46	20	48
45	48	41	27

Write commands for:

P.T.O.

(i) Putting data into a variable x.

(ii) Creating a box plot of x

(iii) Creating a scatter plot of x

(iv) Creating a stem and leaf plot of x

(v) Creating a normal probability plot of x.

[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8119A** **HC**

Unique Paper Code : 62353505

Name of the Course : **B.A. (Prog.) Mathematics: SEC**

Name of the Paper : Statistical Software- R

Semester : V

Time : 2 Hours ***Maximum Marks : 38***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

(c) **All** commands should be written in software R.

1. Do any **five** of the following : 1×5=5

State whether the following statement are ***true or false*** :

(i) Command to evaluate sin (60^0) is

$$\sin\left(\frac{\pi}{60}*180\right)$$

P.T.O.

(ii) setwd () command is used to find the default path of the files saved

(iii) If a = 2 + 5 and b = e^5, then a + b = c gives 7 + e^5.

(iv) savehistory (file =' Rhisotry') is same as loadhistory (file =' Rhistory') command.

(v) If datag is a ten item vector then datag [-3] command show only third items.

(vi) The commands seq_along (dataname) and se(along = dataname) gives the same output

2. Do any **five** of the following. 1×5=5

Fill in the blanks.

(i) .... command is used to verify if a given object "X" is a matrix data object. (is. matrix (X)/class(X)).

(ii) In two digit number, ... digit represent the stem value in stem-and-leaf plot. (ones/tens) (iii).............. command is used to find the row sums of any data frame object "Z". (row Sums()/rowsums()).

(iv) hist() command is used for .... (history, histogram).

(v) For calling function, we use.............. bracket ((),[]).

(vi) To rearrange data, we use .... command (sort().order()).

3. (a) (i) Using scan command enter the following data : 2×8=16

Mon, Tue, Wed, Thus, Fri, Sat, Sun

(ii) Write command to read a csv file.

(b) Write a command for the following :

(i) To list all the elements starting with either 'n' or 'j'.

(ii) To remove all the variables containing 'I' as the last alphabet.

(c) Identify the errors in the command and correct them

Seq [from = 1, To = 10, by = 2]

 P.T.O.

(d) (i) What will be the class of the resulting vector if you concatenate a number and NA.

(ii) How will you convert a data frame into a table.

(e) Differentiate between seq(5) and seq along(5) commands.

(f) Consider a matrix X

	Q1	Q2	Q3	Q4
R1	Jan	Apr	Jul	Oct
R2	Feb	May	Aug	Nov
R3	Mar	Jun	Sep	Dec

(i) Write command to change the name of rows with a,b,c and name of columns with A,B,C,D respectively.

(ii) Print all items of 2nd columns.

(g) Rearrange the data in increasing order and draw a stem and leaf plot where data is :

X = 3,5,7,5,3,6,8,.5,4,5,9,7,4

(h) Make a score data file

81	81	96	77
95	98	73	83
92	79	82	93
80	86	89	60
79	62	74	60

Draw a stem leaf plot

Do any **four** of the following :

3×4=12

(a) (i) How to make a comment in R ?

(ii) Create a vector

x : 12, 7, 3, 4.2, 18, -21, NA.

(iii) Find the mean and median of vector x.

(iv) Find mean of vector x by dropping NA values.

(v) Find the quantile of vector x.

(b) (i) Create data strings :

x : 3 7 9 5

labels : Landon New York Singapore Mumbai.

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7336**

Unique Paper Code : **32355101** **HC**

Name of the Paper : **Calculus**

Name of the Course : **Generic Elective for Honours : Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *five* questions from each of the *three* Sections.

Each question is of *five* marks.

Section I

1. Use $\varepsilon - \delta$ definition to show that :

$$\lim_{x \to 4} (9 - x) = 5.$$

P.T.O.

2. Find the asymptotes for the curve :

$$f(x) = \frac{x^2 - 3}{2x - 4}.$$

3. Find the Linearization L(x) of $f(x)$ at $x = a$ where :

$$f(x) = x + \frac{1}{x} \text{ at } a = 1.$$

4. For $f(x) = (x - 2)^3 + 1$

(i) Find the intervals on which f is increasing and the intervals on which f is decreasing.

(ii) Find where the graph of f is concave up and where it is concave down.

5. Use L'Hôpital's rule to find :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right).$$

6. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

7. Find the length of the curve :

$$x = t^3,\ y = \frac{3t^2}{2},\ 0 \le t \le \sqrt{3}.$$

Section II

8. State Limit comparison test. Using the limit comparison test, show that :

$$\int_1^{\infty} \frac{3dx}{e^x + 5} \text{ converges.}$$

9. Identify the symmetries of the curve and then sketch the graph of :

$$r^2 = \cos\theta.$$

16. Find the derivative of the function f at p_0 in the direction of $\vec{A}$ where $f(x, y, z) = 3e^x \cos yz$, $p_0(0, 0, 0)$, $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$.

17. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces : $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$, $x^2 + y^2 + z^2 = 11$

Point : $(1, 1, 3)$.

18. Find equations for the :

(*a*) Tangent plane and

(*b*) Normal line at the point p_0 on the given surface :

$f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $p_0(1, 2, 4)$.

19. Find the absolute maximum and minimum value of :

$$f(x, y) = 2 + 2x + 2y + x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

20. If

$$f(x, y) = x^2 - y^2, \quad g(x, y) = 3xy + y^2x,$$

show that :

(i) $\nabla(fg) = f\nabla g + g\nabla f$

(ii) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$.

21. If $w = x \sin y + y \sin x + xy$, show that $w_{xy} = w_{yx}$.

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7467** **HC**

Unique Paper Code : 32355301

Name of the Course : **Generic Elective for Honours : Mathematics**

Name of the Paper : Differential Equations

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions by selecting any **two** parts from each question.

1\. (a) Solve the differential equation by finding an integrating factor:

$$(e^{(x+y)}+ye^{y})dx+(xe^{y}-1)dy=0$$

6.5

(b) Solve the differential equation $y'=5.7y-6.5y^2$. 6.5

(c) Find the orthogonal trajectories of $x=c\sqrt{y}$.

P.T.O.

2. (a) Solve $((3x^2+2x+\sin(x+y))dx+\sin(x+y)dy=0$. 6

(b) Show that x^2 and x^{-2} form a basis of the following differential equation $x^2y''+xy'-4y=0$. Also find the solution that satisfies the conditions $y(1)=11$, $y'(1)=-6$. 6

(c) Find the radius of convergence of the series $\sum_{m=0}^{\infty}\frac{(-1)^m x^{3m}}{8^m}$ 6

3. (a) Find the general solution of the following differential equation using method of variation of parameters $y''+9y=\sec 3x$. 6.5

(b) Use the method of undetermined coefficients to find the solution of the differential equation: $y''+3y'+2.25y=-10\,e^{-1.5x}$, $y(0)=1, y'(0)=0$. 6.5

(c) Find a homogenous linear ordinary differential equation for which two functions x^{-3} and $x^{-3}\ln x$ $(x>0)$ are solutions. Also show the linear independence by considering their Wronskian. 6.5

4. (a) Find the general solution of the linear partial differential equation

$x(y^2-z^2)u_x+y(z^2-x^2)u_y+z(x^2-y^2)u_z=0.$ 6

(b) Find the general solution of the differential equation: $(x^2D^2+6xD+6I)y=0.$

Where $D=\frac{d}{dx}$ 6

(c) Find the particular solution of the linear system that satisfies the stated initial conditions:

$\frac{dy_1}{dt}=y_1+y_2,\quad y_1(0)=1$

$\frac{dy_2}{dt}=4y_1+y_2,\quad y_2(0)=6.$ 6

5. (a) Find the power series solution of the following differential equation, in powers of x

$y''-y'=0.$ 6.5

(b) Find the solution of the Cauchy problem: $xu_x+yu_y=xe^{-u}$, with the $u=0$ when $y=x^2$. 6.5

P.T.O.

(c) Reduce the equation: $u_x+xu_y=y$ to canonical form, and obtain the general solution. 6.5

6. (a) Solve the initial-value problem:

$au_x+bu_y=0,\ u(x,0)=\alpha e^{\beta x}$ by the font is different. 6

(b) Reduce the: $u_{tt}-c^2u_{xx}=0, c\neq 0$ where c is a constant, into canonical form and hence find the general solution. 6

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx}+2u_{xy}+u_{yy}=0$$

into canonical form and hence find the general solution. 6

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **8161A** **HC**

Unique Paper Code : 62355503

Name of the Course : **Mathematics : Generic Elective**

Name of the Paper : General Mathematics-I

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions as per directed question wise.

Section - I

1. Write a short note on the life and contributions of any **three** of the following mathematicians :

(a) Galois,

(b) Riemann,

(c) Weierstrass,

(d) Abel,

(e) Laplace

P.T.O.

Section - II

2. Attempt any **six** questions. Each question carries **five** marks.

(a) Define Perfect numbers and Amicable numbers. State the properties of Perfect numbers.

(b) Define the magic square and state properties of Benjamin Franklin's magic square.

(c) Define the Inversion and explain the Fifteen Puzzle.

(d) Find the remainder when

$12345 \times 123456 \times 1234567$ is divided by 11.

(e) Explain continued fraction and express $\frac{221}{41}$ as continued frction.

(f) Define unit fraction and express $\frac{2}{7}$ and $\frac{98}{100}$ as unit fraction .

(g) (i) In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together ?

(ii) Use the Egyptian method of duplation to find 58×93.

Section - III

3. Do any **three** questions. Each question carries **six** marks.

(a) If $A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ 3 & 5 \end{pmatrix}$, find a and b such that $AB = BA$.

(b) If $A = \begin{pmatrix} 6 & 2 & -1 \\ 4 & 3 & 1 \\ 1 & -2 & 0 \end{pmatrix}$, then calculate A^3 ?

(c) Express the matrix $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$ as sum of skew-symmetric and symmetric matrix.

(d) Find the inverse of the matrix $\begin{pmatrix} -4 & 7 & 6 \\ 5 & -5 & -4 \\ -2 & 4 & 3 \end{pmatrix}$, if it exist.

P.T.O.

4. Do any **two** questions. Each question is of **six** marks.

(a) $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 6 & 1 \\ 3 & 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 2 & -1 \\ 6 & 4 & 6 \end{pmatrix}$, then is AB = BA ? Verify.

(b) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -4 \\ 1 & 2 \end{pmatrix}$, then is determinant (AB) = determinant (BA) Verify.

(c) Use Cramer's rule to solve the system :

$5x_1 - 3x_2 - 10x_3 = -9$

$2x_1 + 2x_2 - 3x_3 = 4$

$-3x_1 - x_2 + 5x_3 \ -1$