This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2002**

Unique Paper Code : **62351201** **GC-4**

Name of the Paper : **Algebra**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set V of all ordered triples of real numbers of the form $(x, y, 0)$ under the operations $\oplus$ and $\odot$ defined by :

$$(x, y, 0) \oplus (x', y', 0) = (x + x', y + y', 0)$$

$$c \odot (x, y, 0) = (cx, cy, 0)$$

forms a vector space over **R**. 6

P.T.O.

2002

(b) Let V be a vector space with operators $\oplus$ and $\odot$. Let W be a non-empty subset of V. Prove that W is a subspace of V if and only if the following conditions hold :

(i) If u, v are vectors in W, then $u \oplus v$ is in W

(ii) If c is any real number and u is any vector in W, then $c \odot u$ is in W. 6

(c) Define basis of a vector space V. Check whether the set $\{(3, 2, 2), (-1, 2, 1), (0, 1, 0)\}$ forms basis for R^3 ? 6

2. (a) Reduce the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence determine its rank. 6½

(b) Verify that the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies its characteristic equation and hence obtain A^{-1}. 6½

(c) Solve the system of linear equations : 6½

$$x - 3y + z = -1$$

$$2x + y - 4z = -1$$

$$6x - 7y + 8z = 7.$$

3. (a) If

$$\cos\alpha + 2\cos\beta + 3\cos\gamma = 0 = \sin\alpha + 2\sin\beta + 3\sin\gamma,$$

prove that :

$$\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha + \beta + \gamma)$$

$$\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma).$$ 6

(b) Prove that : 6

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}.$$

(c) Sum the series :

$$\cos\theta\sin\theta + \cos^2\theta\sin 2\theta + \ldots\ldots + \cos^n\theta\sin n\theta,$$

where $\theta \neq k\pi$. 6

4. (*a*) Find the rational roots of the equation : 6½

$$x^4 - x^3 - 19x^2 + 49x - 30 = 0.$$

(*b*) Solve the equation :

$$27x^3 + 42x^2 - 28x - 8 = 0,$$

the roots being in G.P. 6½

(*c*) If α, β, γ, be the roots of the equation : 6½

$$x^3 + px^2 + qx + r = 0, \ (r \neq 0).$$

find the value of :

(*i*) $\Sigma(\beta + \gamma)^2$ 6.

(*ii*) $\Sigma\alpha^{-2}$.

5. (*a*) Let n be a positive integer. Prove that the congruence class $[a]_n$ has a multiplicative inverse in Z_n if and only if $(a, n) = 1$. 6½

(b) Consider the following permutations in S_7 : 6½

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$$

and $$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Compute the following products :

(i) $\tau^{-1}\sigma\tau$

(ii) $\tau^2\sigma$.

(c) Prove that the set Q^+ of all positive rational numbers is an abelian group under the binary operation $*$ defined by $a * b = \dfrac{ab}{3}$. 6½

6. (a) Let $G = GL_2(R)$. Prove that :

$$D = \left\{ \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}, \; ad \neq 0 \right\}$$

is a subgroup of G. 6

(*b*) Prove that the set $S = \{0, 2, 4\}$ is a subring of the ring Z_6 of integers modulo 6.

(*c*) Prove that the rigid motions of an equilateral triangle yields the group S_3.

This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : 3019

Unique Paper Code : 62354443

GC-4

Name of the Paper : Analysis

Name of the Course : B.A. (Prog.) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

All questions are compulsory.

Marks are indicated against each part of the questions.

1. (*a*) Which of the following sets are bounded below, which are bounded above and which are neither bounded below nor bounded above.

(*i*) $\{-1, -2, -3, -4, \ldots\ldots\ldots, -n, \ldots\ldots.\}$

(*ii*) $\{-1, 2, -3, 4, \ldots\ldots\ldots, (-1)^n n, \ldots\ldots.\}$

(*iii*) $\{2, \frac{3}{2}, \frac{4}{3}, \ldots\ldots\ldots, \left(\frac{n+1}{n}\right), \ldots\ldots.\}$

(*iv*) $\{3, 3^2, 3^3, \ldots\ldots\ldots, 3^n, \ldots\ldots.\}$

(v) $\{1, \frac{1}{2}, \frac{1}{2^2}, \ldots\ldots\ldots, \frac{1}{2^n} \ldots\ldots\}$

(vi) $\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots\ldots\ldots, \frac{(-1)^n}{2}, \ldots\ldots\}$ 6.5

(b) Define supremum and infimum of a non-empty bounded set. Suppose A and B are two non-empty subsets of **R** satisfying the property :

$$a \leq b, \forall a \in A \text{ and } \forall b \in B.$$

Prove that :

$$\sup(A) \leq \inf(B).$$ 6.5

(c) State Bolzano Weirstrass theorem for sets. Show by an example that the conditions in this theorem cannot be relaxed. 6.5

2. (a) Prove that every continuous function on a closed interval is bounded. 6.5

(b) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0, \infty[$. 6.5

(c) Define an open set and prove that the union of an arbitrary family of open sets is an open set. 6.5

3. (a) Define a convergent sequence and a bounded sequence. Show that the sequence $< a_n >$ defined by :

$$a_n = (-1)^n, \forall n$$

is bounded but not convergent. 6.5

(b) Show that the sequence $< a_n >$ defined by :

$$a_1 = 1, \quad a_{n+1} = \sqrt{2 + a_n}, \ \forall \ n \geq 1$$

is bounded and monotonic. Also find $\lim_{n \to \infty} a_n$. 6.5

(c) State and prove Cauchy convergence criterion for sequences. 6.5

4. (a) Prove that every monotonically increasing and bounded above sequence converges. 6

(b) If $< a_n >$ and $< b_n >$ are sequences of real numbers such that :

$$\lim_{n \to \infty} a_n = a, \quad \lim_{n \to \infty} b_n = b$$

then prove that :

$$\lim_{n \to \infty} (a_n + b_n) = (a + b).$$ 6

(c) Show that :

$$\lim_{n \to \infty} \frac{2^{3n}}{3^{2n}} = 0.$$ 6

5. (*a*) State and prove Cauchy's *n*th root test for an infinite series. 6

(*b*) Test for convergence the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+5}$

(*ii*) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$. 6

(*c*) State (without proof) D'Alambert's ratio test for an infinite series. Test for convergence the series :

$$\frac{1}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\frac{4!}{5^4}+\ldots\ldots\ldots\ldots.$$ 6

6. (*a*) Define an alternating series. State Leibnitz's test for an alternating series. Test for convergence the series :

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\ldots\ldots\ldots\ldots.$$ 6

(*b*) Test for convergence the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin n\alpha}{n^3}$, α being real

(*ii*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$. 6

(*c*) Prove that every monotonically increasing function f on $[a, b]$ is Riemann integrable on $[a, b]$. 6

'his question paper contains 14 printed pages.]

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B.A. Prog. / III **G**

Paper Code : C-155

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

ime : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :- The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. A). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **six** *questions in all selecting* **two** *parts from each question.* **Unit I** *and* **Unit II** *are compulsory and contain* **four** *questions. In* **Unit III** *choose* **one** *of* **III(1), III(2), ..., III(5)** *and attempt* **both** *questions from the same. Marks are indicated against each question.*

Use of scientific calculator is allowed.

P.T.O.

Unit I

(Real Analysis)

1. (a) Prove that the intersection of a finite number of open sets is open. What happens if the family consists of infinite number of open sets ? Justify your answer. (6)

(b) Define limit point of a set. Prove that a finite set has no limit point. (6)

(c) Discuss the continuity at x = 3 of the function f defined by

$f(x) = x - [x] \ \forall \ x \geq 0$, where [x] is the greatest integer $\leq x$. (6)

2. (a) Prove that sequence $<a_n>$ defined by the relation

$$a_1 = 1,\ a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots\dots + \frac{1}{(n-1)!}\ (n \geq 2),$$

converges. (7)

(b) Test the convergence of any two of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^p}$

(ii) $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\ldots\ldots\ldots\ldots$

(iii) $\sum\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ (7)

(c) State Leibnitz Test for the convergence of an alternating series. Test the convergence and the absolute convergence of the series :

$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\ldots\ldots\ldots$ (7)

3. (a) Prove that the function f(x) defined on [0,1] as :

$$f(x)=\begin{cases}1 & \text{if x is rational}\\ 0 & \text{if x is irrational}\end{cases}$$

is not Riemann integrable. (6)

(b) Show that the integral

$$\int_0^\infty e^{-x}x^{n-1}\,dx$$

converges if and only if $n>0$. (6)

(c) (i) Prove that the series

$$\frac{\sin x}{\sqrt{1}} + \frac{\cos 2x}{\sqrt{2}} + \frac{\sin 3x}{\sqrt{3}} + \frac{\cos 4x}{\sqrt{4}} + \ldots\ldots\ldots$$

is not a Fourier series.

(ii) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

(6

Unit II

(Computer programming)

4. (a) (1) What are the two forms of floating constants in C/C++ ?

(2) What will be the value of 15.9/3 and 19% 6 in C/C++ ?

(3) How do the computer detects whether an identifier used in a program is a constant or a variable or an array ? Support your answer with example, for all three. (6½)

(b) (1) What will be the value of b if a = 10 initially when the following statement is executed ?

1. b = ++a + ++a & 2. b = ++a + a++ (6½)

(2) Give short notes on the iterative structures in C/C++, with examples. (6½)

(c) Write a program to calculate the area of a circle, a rectangle & a triangle depending upon user's choice, using switch structure. (6½)

Unit III(1)

(Numerical Analysis)

(a) Consider the equation

$$f(x) = x^4 - 3x^2 + x - 10 = 0$$

(1) Find the interval of unit length which contains smallest positive root of the equation.

(2) Perform two iterations by the Bisection Method, taking the initial interval as considered in part (1).

(3) Taking the midpoint of the last interval of part (2) as the initial approximation, obtain the root, correct to two places of decimal, by the Newton Raphson Method. (6)

(b) Compare the Bisection method with Newton Raphson method for solving an equation.

Also do the comparison of Gauss Seidel Method with the Gauss Jacobi method, for solving the system of Linear equations, mentioning the advantages of one method over the other.

(6)

(c) Find the inverse of the coefficient matrix of the system

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

by Gauss Jordan method with partial pivoting & hence solve the system. (6)

6. (a) Find the unique interpolating polynomial P(x) of degree 2 or less, which interpolates f(x) at the points $x = 0, 1, 4$ such that $f(0) = 1$, $f(1) = 27$, $P(4) = 64$ by

(1). Lagrange's method

(2) Newton's Divided Difference method

Hence evaluate f(3). (6)

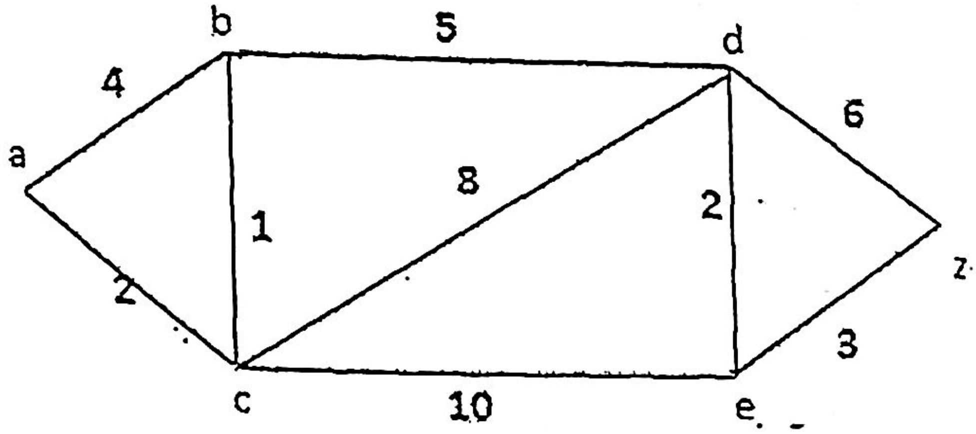
(b) Determine an appropriate step size to be used in the tabulation of $f(x) = (1 + x)^6$ in the interval [0, 1], so that the truncation error for linear interpolation is to be bounded by 5×10^{-5}. (6)

(c) Evaluate $\int_0^1 \frac{1}{1+x}\,dx$, correct to three decimal places by Trapezoidal rule and Simpson's $1/3^{rd}$ rule with $h = 0.5$. Also determine which method yields more accurate result & Why ? (6)

Unit III(2)

(Discrete Mathematics)

5. (a) Find the length of a shortest path between a and z in the given weighted graph (6)



(b) Show that if G is a connected planar simple graph, then G has a vertex of degree not exceeding five. Is K_s planar graph ? (6)

(c) Define the following terms with example :

(i) A directed graph

(ii) A Hamilton circuit

(iii) A Eulerian path.

(6)

6. (a) Write the following Boolean expression :

$$E(x_1, x_2, x_3) = \left(x_1 \wedge x_2 \wedge \overline{x_3}\right) \vee \left(x_1 \wedge \overline{x_2} \wedge x_4\right) \wedge \left(x_2 \wedge \overline{x_3} \wedge \overline{x_4}\right)$$

over the two valued Boolean algebra in conjunctive normal form.

(6)

(b) Draw the switching circuit corresponding to the algebraic expression :

$$a \wedge \left[\left(b \vee \overline{d}\right) \vee \left\{\overline{c} \wedge \left(a \vee d \vee \overline{c}\right)\right\}\right] \vee b$$

(6)

(c) Show that the following statement is a tautology:

$(A \rightarrow B) \rightarrow \left[\left(\overline{A} \rightarrow B\right) \rightarrow B\right]$, where $\overline{A}$ denotes the negation of A.

(6)

Unit-III(3)

(Mathematical Statistics)

5. (a) Show that for any discrete distribution, standard deviation is not less than mean deviation from mean.

(6)

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ? (6)

(c) Two independent variables x_1 and x_2 are with means 5 and 10 and variances 4 and 9. If $u = 3x_1 + 4x_2$ and $v = 3x_1 - x_2$, find the correlation coefficient between u and v. (6)

6. (a) Prove that the recurrence relation for the Poisson distribution with mean λ is $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d\mu_r}{d\lambda}$, where μ_r is the r^{th} moment about the mean. Hence deduce the values of β_1 and β_2. (6)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given that if

$f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t \exp\left(-\frac{x^2}{2}\right)dx$, Then $f(0.496) = 0.9$ and $f(1.405) = 0.42$. (6)

(c) For two variables X and Y, the two regression lines are $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$. If $Var(X) = 9$

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Calculate (i) the mean values of X and Y

(ii) the correlation coefficient between X and Y. (6)

Unit III(4)

(Mechanics)

5 (a) Three forces each equal to 'P' act along the sides of triangle ABC in order, prove that the resultant is:

$$P\left[1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]^{1/2}$$

and find the distance of its line of action from one angular point ,and here it cuts one side of the triangle. (6)

(b) Two weights w_1 and w_2 rest on a rough plane inclined at an angle α to the horizontal and are connected by a string which lies along the line of greatest slope. If μ_1 and μ_2 are their coefficients of friction with the plane and $\mu_1 > \tan\alpha > \mu_2$, prove that, if they are both on the point of slipping, (6)

$$\tan\alpha = \frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2}.$$

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(c) Find the centre of gravity of a quadrant of a circular disc of radius a. (6)

(a) A particle is performing a S.H.M of period T about a centre 'O' and it passes through a point P, where OP = b with velocity v in the direction OP. Prove that the time which elapses before its return to P is

$$\left(\frac{T}{\pi}\right)\tan^{-1}\left(\frac{vT}{2\pi b}\right). \qquad (6)$$

(b) A particle moves under the influence of a centre which attracts with a force :

$$\left(\frac{b}{r^2}+\frac{c}{r^4}\right),$$

'b' and 'c' being positive constants and 'r' the distance from the centre. The particle moves in a circular orbit of radius 'a'. Prove that the motion is stable if and only if, $a^2b > c$. (6)

(c) If v_1 and v_2 be the velocities at the ends of a focal chord of a projectile's path and 'u', the horizontal component of velocity, show that

$$\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2} \qquad (6)$$

Unit III(5)

(Theory of Games)

5. (a) Solve graphically the LPP :

Minimize $Z = 20x_1 + 10x_2$

subject to :

$$x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

(b) Use simplex method to solve following problem :

Maximize $Z = 4x_1 + 10x_2$

subject to : $2x_1 + x_2 \leq 50$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

(c) Verify that the dual of dual is primal for the following LPP :

Maximize : $Z = 2x_1 + 5x_2 + 6x_3$

subject to :

$$5x_1 + 6x_2 - x_3 \leq 3$$

$$-2x_1 + x_2 + 4x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

(6)

(a) Use graphical method to solve the rectangular game whose pay off matrix is :

$$\begin{bmatrix} -2 & -6 \\ -4 & -5 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

(6)

(b) Find the range of values of p & q which will render (2,2) a saddle point of the game :

$$\begin{bmatrix} 0 & 2 & 3 \\ 8 & 5 & q \\ 2 & p & 4 \end{bmatrix}$$

(6)

(c) Reduce the following game to an LPP and hen
solve :

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}$$

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **3051**

Unique Paper Code : **62353424** **GC-4**

Name of the Paper : **Computer Algebra Systems and Related Softwares**

Name of the Course : **B.A. (Prog.) Mathematics Skill Enhancement Course**

Semester : **IV**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 10×1

(*i*) The argument of a function in Mathematica is given by ....

(*ii*) The command .... determines the prime factorization in Mathematica.

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(*iii*) The command .... is used for matrix exponentiatio

in Maxima.

(*iv*) Thr plotting engine that beneath Maxima is called ......

(*v*) In Maple .... breaks the line.

(*vi*) Lines in Maple may be terminated with .... to

suppress the output of the calculation.

(*vii*) The command .... returns the rank of a matrix in

MATLAB/Octave.

(*viii*) MATLAB stands for ....

(*ix*) The command for $\sqrt{10}$ in R is ....

(*x*) In R, the .... function produces box

plots.

2. Answer any *eight* parts from the following : 8×2½

(*i*) Write the commands in R for the following :

(*a*) Put the list of 2, 5, 6, 10, 11, 9, 1, 0, 6, 8, 7

into a variable B.

(*b*) To sort the array B.

(*c*) To find simple standard deviation of B.

(*ii*) Write the command in R to simulate a random sample of 20 items from a normally distributed data that has mean 50 and standard deviation 8.

(*iii*) Write a program in MATLAB/Octave to plot the graph of a circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 2\pi$ with step size 0.01.

(*iv*) Write the output for the following commands :

(*a*) zeros (2, 3)

(*b*) eye (2)

(*c*) ones (3)

(*d*) v = [4; 3; 5]

(*e*) s = [1, 0, 2].

(*v*) Write *two* differences between Maxima and Maple.

(*vi*) Write the use of the following commands in mathematica :

(*a*) PlotLegend

(*b*) Do loop

(*c*) ;

(*vii*) Define and differentiate a function $f(x) = x^2 + \sin x$ in Maxima.

(*viii*) Write a program in Maxima to plot the surface :

$$g(x, y) = \cos x + \sin y$$

for $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

(*ix*) Write a program to plot the surface $z = e^{-(x^2 + y^2)}$ for $-2 \leq x, y \leq 2$ in Maple.

(*x*) Write a command to construct a table of the first 50 prime numbers in Mathematica.

3. Answers any *four* parts from the following : 5×4

(*i*) Write a program in R for the following :

(*a*) Put the data into a variable x :

19	8	6	3	9	1
2	8	8	9	2	9
4	6	3	1	8	3
6	3	7	4	7	2
8	4	8	3	2	9

(*b*) Generate a five number summary of x.

(*c*) Create a box plot of x.

(*d*) Create a stem and leaf plot of x

(*e*) Create a normal probability plot of x.

(ii) Write a program in MATLAB/Octave to solve the system of equations :

$$2y - z + 3t = 20$$

$$2x + y + z - t = 29$$

$$-5x + y + 3t = 12$$

$$7x - 2y + 3z + 4t = 18 .$$

(iii) Write the program in Mathematica, to plot the function :

$f(x) = e^x - \sin x$, $g(x) = x^2 \cos\left(\frac{1}{x^2}\right)$,

$h(x) = x^3 + x - 1$ for $0 \le x \le 3$.

Use the commands for plot legend, color and thickness.

(iv) Explain thru-do Loop in Maxima. Set $c = 1.6 - 0.8i$ and $z = 0$, then write a program in maxima to iterate $f(z) = 2z^2 - c$ ten times.

(v) Write the command in Maple for the following :

(a) Let $A = \begin{pmatrix} -1 & 0 & 3 \\ 2 & 2 & 5 \\ 6 & 3 & 1 \end{pmatrix}$ and

$$B = \begin{pmatrix} 3 & -1 & 1 \\ -2 & 2 & 5 \\ 3 & 6 & 7 \end{pmatrix}$$

(b) Find A + B

(c) Find matrix multiplication of A and B

(d) $\Sigma_{i=1}^{20} (i^2 + i + 1)$

(e) 7^9 mod 8.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 276

Unique Paper Code : 235251 G

Name of the Paper : Algebra

Name of the Course : B.A. (Prog.) Discipline Course

Semester : II

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set :

$$S = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} : x, y, z \in \mathbf{R} \right\}$$

is a vector space over the field **R** with respect to matrix addition and multiplication of a matrix by a scalar. 6

(*b*) Show that the set :

$$W = \{(a_1, a_2, a_3) : a_1 + a_2 + a_3 = 0, a_1, a_2, a_3 \in \mathbf{R}\}$$

is a subspace of $\mathbf{R}^{(3)}$. 6

P.T.O.

(c) Express the vector $v = (3, 1, -4)$ as a linear combination of :

$a = (1, 1, 1)$, $b = (0, 1, 1)$ and $c = (0, 0, 1)$.

Is $S = \{a, b, c\}$ a basis of $\mathbf{R}^{(3)}$? Justify. 6

2. (a) Solve the system of equations : 6.5

$$x + y + z = 2$$

$$x - 3y + 4z = 3$$

$$3x - 8y + 11z = 11$$

(b) Find the rank of the matrix : 6.5

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 1 & 2 & -3 & 1 \\ -1 & 2 & 3 & -1 \end{bmatrix}.$$

(c) Find the characteristic roots of the matrix : 6.5

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

3. (a) Show that : 6

$$\sin^5\theta = \frac{1}{16}\{\sin 5\theta - 5\sin 3\theta + 10\sin\theta\}.$$

(*b*) Sum to n terms : 6

$$\sin\theta + \sin 2\theta + \dots\dots + \sin n\theta .$$

(*c*) Solve the equation : 6

$$z^{10} - z^5 + 1 = 0 .$$

4. (*a*) The sum of two of the roots of the equation 6.5

$$x^3 - 5x^2 - 16x + 80 = 0$$

is zero. Find the roots.

(*b*) Solve the equation 6.5

$$x^3 - 13x^2 + 15x + 189 = 0 ,$$

being given that one of the roots exceeds another by 2.

(*c*) If α, β, γ be the roots (such that sum of any of two of them is non-zero) of the equation : 6.5

$$x^3 + qx + r = 0 ,$$

then find the values of :

(*i*) $\sum \dfrac{1}{\beta+\gamma}$

(*ii*) $\sum \dfrac{\beta^2+\gamma^2}{\beta+\gamma}$.

5. (*a*) Find the multiplicative inverses of the given elements if they exist : [14] in Z_{15} and [35] in Z_{6669}. 6

(*b*) Consider the following permutation in S_7 : 6

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 1 & 7 \end{pmatrix} \text{ and}$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 6 & 4 & 7 & 3 \end{pmatrix}$$

Compute the following products : $\sigma\tau$ and $\tau^2\sigma$.

(*c*) Prove that any group of prime order is cyclic. 6

6. (*a*) Prove that the rigid motion of an equilateral triangle yields the Group S_3. 6.5

(*b*) If A and B are sub-rings of a ring R, show that $A \cap B$ is also a sub-ring of R. 6.5

(*c*) Let

$$G = \{(a, b) : a, b \in R\}$$

equipped with the binary operation * defined by :

$$(a, b) * (c, d) = (a + c, b + d) \text{ for all } a, b, c, d \in R,$$

prove that (G, *) is an abelian Group. 6.5

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 296

Unique Paper Code : 235451

G

Name of the Paper : Mathematics (Analytical Geometry and Applied Algebra)

Name of the Course : B.A. (Prog.) Discipline Course

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

(*b*) Sketch the ellipse :

$$4x^2 + y^2 + 8x - 10y = -13$$

and label the foci, the vertices, and the ends of the minor-axis.

(c) Find the centre vertices, foci and asymptotes of the hyperbola whose equation is :

$$4x^2 - 9y^2 + 16x + 54y - 29 = 0$$

and sketch its graph. 6,6,6

2. (a) Find an equation for the parabola whose axis is $y = 0$ and it passes through the points (3, 2) and (2, –3).

(b) Find on equation of the ellipse whose foci are (1, 2) and (1, 4) and minor-axis is of the length 2.

(c) Find an equation for a hyperbola whose foci are $(0, \pm 5)$ and asymptotes are $y = \pm 2x$. 6,6,6

3. (a) Rotate the coordinate axes to remove the xy-term of the curve

$$31x^2 + 10\sqrt{3}\,xy + 21y^2 - 144 = 0$$

and then name the conic.

(*b*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of 45°. Find an equation of the curve

$$3x'^2 + y'^2 = 6$$

in xy-coordinate system.

(*c*) (*i*) Find the angle between a diagonal of a cube and one of its edges.

(*ii*) Find k so that the vector from the point A(1, −1, 3) to the point B(3, 0, 5) is orthogonal to the vector from A to the point P(k, k, k). 6,6,6

4. (*a*) Find an equation of the sphere that is inscribed in the cube that is centred at the point (−2, 1, 3) and has sides of length 1 that are parallel to the coordinate planes.

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(b) (i) Prove that :

$$\left\|\vec{u}+\vec{v}\right\|^2+\left\|\vec{u}-\vec{v}\right\|^2=2\left\|\vec{u}\right\|^2+2\left\|\vec{v}\right\|^2$$

where $\vec{u}$ and $\vec{v}$ are any two vectors.

(ii) Find the vector component of $\vec{a}$ and $\vec{b}$ and the vector component of $\vec{a}$ orthogonal to $\vec{b}$ where

$$\vec{a}=2\hat{i}-\hat{j}+3\hat{k}\cdot$$

$$\vec{b}=\hat{i}+2\hat{j}+2\hat{k}\cdot$$

(c) (i) Find the volume of the tetrahedron with vertices

P(−1, 2, 0), Q(2, 1, −3), R(1, 0, 1), S(3, −2, 3).

(ii) Find two unit vectors that are normal to the plane determined by the points

A(0, −2, 1), B(1, −1, −2) and C(−1, 1, 0) 6,6,6

5. (*a*) Let L_1 and L_2 be the lines whose parametric equations are :

$$L_1 : x = 4t, \quad y = 1 - 2t, \quad z = 2 + 2t$$

$$L_2 : x = 1 + t, \quad y = 1 - t, \quad z = -1 + 4t.$$

Find parametric equations for the line that is perpendicular to L_1 and L_2 and passes through their point of intersection.

(*b*) (*i*) Find the parametric equations of the line that passes through (–1, 2, 4) and is parallel to

$$3\hat{i} - 4\hat{j} + \hat{k}.$$

Also find the intersection of the line with *xy*-plane.

(*ii*) Find an equation of the plane through the point (–1, 4, 2) that contains the line of intersection of the planes :

$$4x - y + z - 2 = 0 \text{ and}$$

$$2x + y - 2z - 3 = 0.$$

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(c) (i) Show that the line

$$x = -1 + t,$$

$$y = 3 + 2t,$$

$$z = -t,$$

and the plane

$$2x - 2y - 2z + 3 = 0$$

are parallel and find the distance between them.

(ii) Find the equation of the plane through the points,

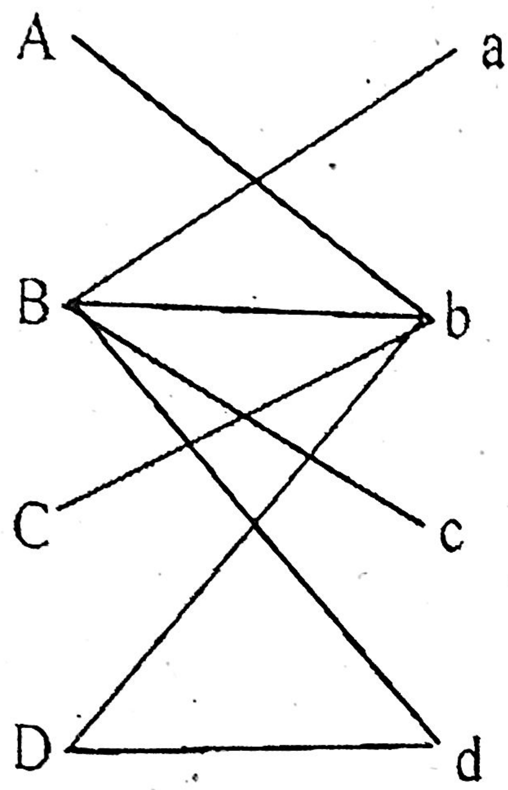
$$P_1(-2, 1, 4),\ P_2(1, 0, 3)$$

that is perpendicular to the plane :

$$4x - y + 3z = 2.$$ 7,7,7

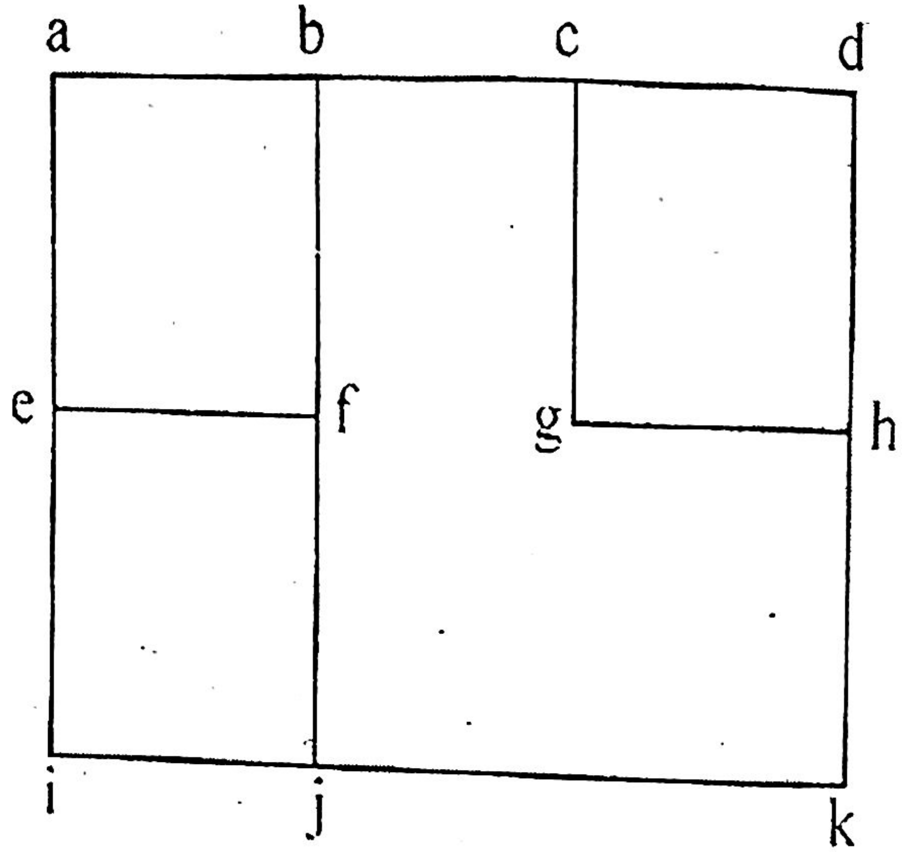
(a) A supermarket wishes to test the effect of putting cereal on four shelves at different heights. Show show to design such an experiment lasting four weeks and using four brands of cereal.

(b) Find a matching or explain why none exists for the following graph :



(c) What are the other sets of 2 edges whose removal disconnects the graph in the following figure besides

(a, b), (a, e) and (c, d), (d, h) ? Either produce other or give an argument why no other exist. 6½,6½,6½



This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **297**

Unique Paper Code : **235451** G

Name of the Paper : **Mathematics (Analytical Geometry and Applied Algebra)**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Sketch the parabola :

$$y = 4x^2 + 8x + 5$$

and label the focus, vertex and directrix.

(*b*) Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

(c) Find the centre, vertices, foci and asymptotes of the hyperbola whose equation is :

$$16x^2 - y^2 - 32x - 6y = 57$$

and sketch its graph. 6,6,6

2\. (a) Find an equation for the parabola that has its vertex at (1, 2) and its focus at (4, 2). Also sketch its rough graph showing the reflection property of parabola at the point (4, –4).

(b) Find an equation of the ellipse whose foci are (2, 1) and (2, –3) and the length of its major axis is 6.

(c) Find an equation for the hyperbola with vertices (2, 4) and (10, 4) and whose foci are 10 units apart. 6,6,6

3\. (a) Identify and sketch the curve $xy = 1$.

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of $\theta = 60°$ and then find an equation of the curve :

$$\sqrt{3}xy + y^2 = 6$$

in $x'y'$-coordinates.

(c) (i) Find the component form of $\vec{v}+\vec{w}$ and $\vec{v}-\vec{w}$ in 2-space, given that :

$$\|\vec{v}\| = 1, \|\vec{w}\| = 1, \vec{v}$$

makes an angle of $\frac{\pi}{6}$ with the positive x-axis, and $\vec{w}$ make an angle of $\frac{3\pi}{4}$ with the positive x-axis.

(ii) Find $\vec{u}$ and $\vec{v}$ if

$$\vec{u} + 2\vec{v} = 3\hat{i} - \hat{k} \text{ and}$$

$$3\vec{u} - \vec{v} = \hat{i} + \hat{j} + \hat{k}.$$ 6,6,6

(a) A sphere S has centre in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3-\sqrt{3}$ units. What is the equation of the sphere ?

(b) (i) Show that two non-zero vectors $\vec{v_1}$ and $\vec{v_2}$ are orthogonal if and only if their direction cosines satisfy :

$$\cos\alpha_1 \cos\alpha_2 + \cos\beta_1 \cos\beta_2 + \cos\gamma_1 \cos\gamma_2 = 0.$$

(ii) Find two unit vectors in 2-space that make an angle of 45° with $4\hat{i} + 3\hat{j}$.

(c) (i) Find the area of the triangle that is determined by the points

$$P_1(2, 2, 0),\ P_2(-1, 0, 2) \text{ and } P_3(0, 4, 3).$$

(ii) Find two unit vectors that are parallel to yz-plane and are orthogonal to the vector $3\hat{i} - \hat{j} + 2\hat{k}$.

6,6,6

5. (a) Let L be the line whose parametric equations are :

$$L : x = 2t, \quad y = 1 - t, \quad z = 2 + t.$$

Find parametric equations of the line that contains the point P(0, 2, 1) and intersects the line L at a right angle.

(b) (i) Let L_1 and L_2 be two lines whose parametric equations are :

$$L_1 : x = 2 - t, \quad y = 2t, \quad z = 1 + t$$

$$L_2 : x = 1 + 2t, \quad y = 3 - 4t, \quad z = 5 - 2t.$$

Show that L_1 and L_2 are parallel and find the distance between them.

(ii) Find the distance between the given parallel planes :

$$-2x + y + z = 0$$

$$6x - 3y - 3z - 5 = 0$$

(c) (i) Find an equation of the sphere with centre (2, 1, −3) that is tangent to the plane :

$$x - 3y + 2z = 4.$$

(ii) Find the equation of the plane through (1, 2, −1) that is perpendicular to the line of intersection of the planes :

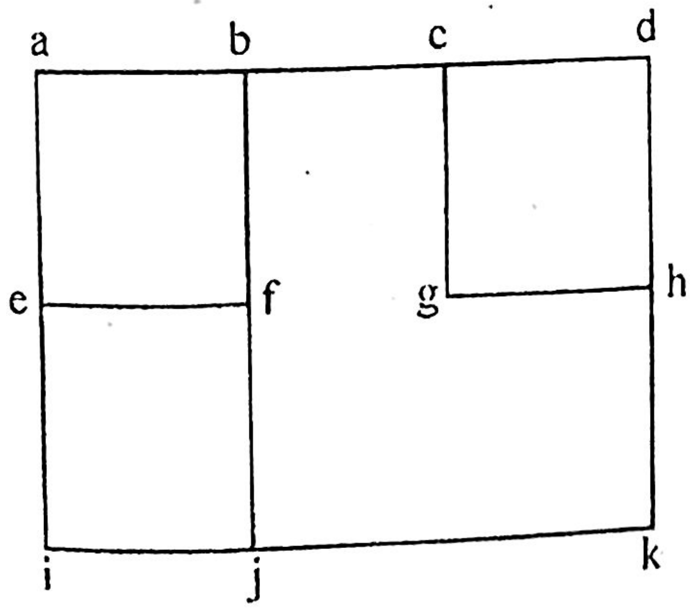
$$2x + y + z = 2 \quad \text{and}$$

$$x + 2y + z = 3.$$

7,7,7

6. (a) Construct a Latin square of order 5 on $\{0, 1, 2, 3, 4\}$.

(b) In the following figure find all sets of three corners that have all 11 corners under serveillance. Give a careful logical analysis.



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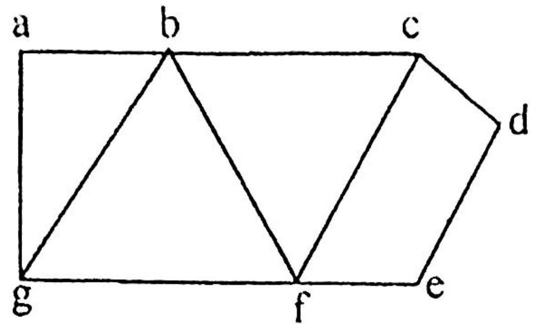
(c) In the following figure find :

(i) All sets of two vertices whose removal disconnects the graph.

(ii) All sets of two edges whose removal disconnects the graph.



6.5,6.5,6.5

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **347**

Unique Paper Code : **235651** **G**

Name of the Paper : **Numerical Analysis and Statistics**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

All questions are compulsory.

Use of scientific calculator is allowed.

Candidate can ask for log/statistical table.

1. (*a*) (*i*) Perform three iterations by Bisection method to obtain the smallest positive root of the equation :

$$f(x) = x^3 - x - 4 = 0.$$

(*ii*) If a root of $f(x) = 0$ lies in the interval (a, b), then what is minimum number of iterations required when the permissible error is ε. 6

(*b*) A real root of the equation :

$$f(x) = x^3 - 5x + 1 = 0.$$

lies in the interval (0, 1). Perform four iterations of Secant method to obtain this root. 6

(*c*) Perform five iterations by Newton-Raphson method to find the root of $N^{1/2}$, where N = 17. Take initial approximation $x_0 = 3$. 6

2. (*a*) Consider the system of equations :

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where a is a real constant. For which values of a the Gauss-Seidel method converges. 6

(b) Solve the following system of equations :

$$2x_1 + x_2 + x_3 - 2x_4 = -10$$

$$4x_1 + 0x_2 + 2x_3 + x_4 = 8$$

$$3x_1 + 2x_2 + 2x_3 + 0x_4 = 7$$

$$x_1 + 3x_2 + 2x_3 - x_4 = -5$$

Using the Gauss-elimination method with partial-pivoting. 6

(c) For the following system of equations :

$$-3x_1 + x_2 + 0x_3 = -2$$

$$2x_1 - 3x_2 + x_3 = 0$$

$$0x_1 + 2x_2 - 3x_3 = -1$$

(i) Show that Jacobi iteration scheme converges.

(ii) Starting with $X^0 = [0, 0, 0]^T$, iterate three times. 6

3. (*a*) Find the unique polynomial of degree 2 or less such that :

$$f(1) = 1, f(3) = 27, f(4) = 64,$$

using Lagrange interpolating formula. Estimate $f(2)$. 6½

(*b*) For the following data :

$$f(0) = 1, f(1) = 14, f(2) = 15,$$

$$f(4) = 5, f(5) = 6, f(6) = 19$$

Obtain the polynomial using Newton divided difference interpolation. Estimate $f(3)$. 6½

(*c*) If

$$f(x) = 1/x,$$

find the divided difference $f[x_1, x_2, x_3, x_4]$. 6½

4. (*a*) Calculate the coefficient of correlation from the following observations : 6

X	Y
2.52	550
2.49	610

2.47	730
2.42	870
1.69	880
3.43	930
4.72	400

(*b*) Determine the line of regression of Y on X for the following data : 6

X	Y
65	67
66	68
67	65
67	68
68	72
69	72
70	69
72	71

(c) Let the pmf $p(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If

$$p(0) = \frac{1}{4} \text{ and if } E(X) = \frac{1}{4},$$

determine $p(-1)$ and $p(1)$. 6

5. (a) Let X have the pdf

$$f(x) = \frac{1}{x^2}, \ 1 < x < \infty,$$

zero elsewhere. Show that E(X) does not exist. 6½

(b) Let X be a random variable such that

$$E[(X - b)^2]$$

exists for all real b. Show that

$$E[(X - b)^2]$$

is minimum when $b = E(X)$. 6½

(c) Determine the mode of normal distribution. 6½

6. (a) In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of error per page,

find the probability that a random sample of pages will contain no error.

(Given, $e^{-0.75} = 0.473$). 6½

(*b*) If X is a normal variate with mean 30 and S.D. 5. Find the probabilities that : 6½

(*i*) $26 \leq X \leq 40$

(*ii*) $X \geq 45$. 6½

(*c*) Determine the moment generating function of Binomial distribution. 6½

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2447**

Unique Paper Code : **32355202** **GC-4**

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective : Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If x and y are vectors in $\mathbf{R}^n$, then prove that :

(*i*) $\|x+y\|^2 = \|x\|^2 + \|y\|^2$ if and only if $x \cdot y = 0$, and

(*ii*) $x \cdot y = \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2\right)$. 3+3

(*b*) Let x and y be non-zero vectors in $\mathbf{R}^n$, then prove that :

$$\|x+y\| = \|x\| + \|y\|$$

if and only if $y = cx$, for some $c > 0$. 6

P.T.O.

(*c*) Using Gauss-Jordan method, find the complete solution set for the following system of homogeneous linear equations : 6

$$4x_1 - 8x_2 - 2x_3 = 0$$
$$3x_1 - 5x_2 - 2x_3 = 0$$
$$2x_1 - 8x_2 + x_3 = 0.$$

2. (*a*) Find the reduced row echelon form matrix B of the following matrix : 4+2½

$$A = \begin{pmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{pmatrix}$$

and then give a sequence of row operations that converts B back to A.

(*b*) Find the characteristic polynomial and eigenvalues of the matrix : 4+2½

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 2 & -4 \\ 3 & -4 & 7 \end{pmatrix}.$$

Is A diagonalizable ? Justify.

(c) Let V be the set $\mathbf{R}^2$ with operations addition and scalar multiplication for x, y, w, z and a in $\mathbf{R}$ defined by :

$$[x, y] \oplus [w, z] = [x + w - 2, y + z + 3], \text{ and}$$

$$a \odot [x, y] = [ax - 2a + 2, ay + 3a - 3].$$

Prove that V is a vector space over $\mathbf{R}$. Find the zero vector in V and the additive inverse of each vector in V. 4+2½

3. (a) Prove that the set $S = \{[3, 1, -1], [5, 2, -2], [2, 2, -1]\}$ is linearly independent in $\mathbf{R}^3$. Examine whether S forms a basis for $\mathbf{R}^3$? 4+2

(b) Find a basis and the dimension for the subspace W of $\mathbf{R}^3$ defined by : 6

$$W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}.$$

(c) Let $S = \{[1, 2], [0, 1]\}$ and $T = \{[1, 1], [2, 3]\}$ be two ordered bases for $\mathbf{R}^2$. Let $v = [1, 5]$. Find the coordinate vector $[v]_S$ and hence find $[v]_T$ using the transition matrix $Q_{T \leftarrow S}$ from S-basis to T-basis. 3+3

4. (*a*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 4 \\ 2 & -1 & 7 \end{pmatrix}, \; b = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

has a solution or not. 6

(*b*) Suppose $L : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation with $L([1, -1, 0]) = [2, 1]$, $L([0, 1, -1]) = [-1, 3]$ and $L([0, 1, 0]) = [0, 1]$. Find $L([-1, 1, 2])$. Also give formula for $L([x, y, z])$, for any $[x, y, z] \in \mathbf{R}^3$. 4+2

(*c*) Let $L : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear operator given by $L([x, y]) = [2x - y, x - 3y]$. Find the matrix for L with respect to the basis $\{[4, -1], [-7, 2]\}$ using the method of similarity. 6

5. (*a*) Consider the linear transformation $L : P_2 \to \mathbf{R}$ defined by :

$$L(p(x)) = \int_0^1 p(x)\, dx,$$

where P_2 is the vector space of polynomials of degree 2 or less. Show that L is onto but not one-to-one.

(b) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $2x + y + z = 0$. Find the minimum distance from the point $P(-6, 10, 5)$ to W. 6

(c) Find a least squares solution for the linear system $Ax = b$, where : 6

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 1 \end{pmatrix}, \; b = \begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix}.$$

(a) Use the similarity method to show that a rotation about the point $(1, -1)$ through an angle $\theta = 90°$, followed by a reflection about the line $x = 1$ is represented by

the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$. 6½

(b) Let $L : P_2 \to \mathbf{R}^2$ be the linear transformation given by :

$$L(p(x)) = [p(1), p'(1)],$$

where P_2 is the vector space of polynomials of degree 2 or less. Find a basis for ker(L) and a basis for range(L), and also verify the dimension theorem. 4+2½

(c) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 3x - y + 4z = 0\}$ of $\mathbf{R}^3$, find the orthogonal complement $W^{\perp}$ and veirfy that $\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3)$. 4+2½

This question paper contains 6 printed pages.]

Your Roll No...............

r. No. of Question Paper : 2744 GC-4

nique Paper Code : 32355444

ame of the Paper : Elements of Analysis

ame of the Course : **Mathematics : Generic Elective for Honours**

emester : IV

uration : 3 Hours Maximum Marks : 75

istructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

All questions are compulsory.

Attempt any **two** parts from each question.

(a) Define a denumerable set. Show that the set $\mathbb{Z}$ of all integers is denumerable. (7.5)

(b) Define supremum and infimum of a non empty subs of R. Find the supremum and infimum of each of th following sets.

(i) $A = \left\{ 1 - \frac{(-1)^n}{n} : n \in N \right\}$

(ii) $B = \left\{ \frac{1}{n} : n \in N \right\}$ (7.

(c) (i) For any two real numbers x and y prove that

$$|x + y| \leq |x| + |y| .$$

(ii) Define cluster point of a set $A \subseteq R$. Show th the finite set A = {1,2} has no cluster points. (7.

2. (a) Use the definition of the limit of a sequence to sho that

(i) $\lim_{n \to \infty} \left(\frac{3n+2}{n+1} \right) = 3$

(ii) $\lim_{n \to \infty} \left(\frac{1}{n^2+1} \right) = 0$ (7.5

(b) State Monotone Convergence Theorem. Hence prove that the sequence $\langle x_n \rangle$ defined by

$$x_1 = 1,\ x_{n+1} = \sqrt{2x_n},\quad \text{for } n \in N$$

is convergent. (7.5)

(c) Define Cauchy sequence. Show that the sequence $\langle x_n \rangle = \langle 1 + (-1)^n \rangle$ is not a Cauchy sequence. (7.5)

(a) If the series $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim_{n\to\infty} a_n = 0$. Is the converse true ? Justify. (6.5)

(b) Show that the series $\sum_{n=1}^{\infty} r^n$ converges if and only if $|r| < 1$. (6.5)

(c) State limit comparison test for positive term series. Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ (6.

4. (a) Define an absolutely convergent and a conditiona convergent series. Prove that every absolute convergent series in R is also convergent.

(b) State Ratio test and hence prove that the series $\sum_{n=1}^{\infty}$

is convergent. (6

(c) Test the convergence and absolute convergence of t following series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

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(a) Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{(2n)!}.$$ (5)

(b) Derive the power series expansion for $f(x) = \sin x$. (5)

(c) Prove the identity

$$C(x+y) = C(x)C(y) - S(x)\,S(y), \quad \text{for all } x, y \in R$$

where $S(x)$ and $C(x)$ denote the sine and cosine functions respectively. (5)

(a) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Also check the convergence at the end points of the interval. (5)

(b) State integration theorem of power series. Show by integrating the series for $\frac{1}{1+x}$ that if $|x| < 1$, then

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n.$$ (5)

(c) Define exponential function $E(x)$ as a sum of power series. Find the domain of $E(x)$ and show that

$$E(x+y) = E(x).E(y), \text{ for all } x, y \in R.$$