[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1962 **GC-3** **Your Roll No...............**

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two parts** from each question.

1. (a) Discuss the existence of the limit of the function $f(x) = |2x - 1|$ at $x = \frac{1}{2}$. (6)

(b) Examine the continuity of the function at $x = 0$ for

$$f(x) = \begin{cases} x\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Also state the kind of discontinuity, if any. (6)

(c) Examine the following function for differentiability at $x = 0$ and $x = 1$:

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 1/x & x > 1 \end{cases}$$ (6)

2. (a) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}. \qquad (6)$$

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + 2n^2 y_n = 0. \qquad (6)$$

(c) If $z = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z. \qquad (6)$$

3. (a) If the tangent to the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ cuts off intercept p and q from the axis of x & y respectively. Show that $\frac{p}{a} + \frac{q}{b} = 1$. (6)

(b) Find the point where the tangent to the curve $y = x^2 - 3x + 2$ is perpendicular to the line $y = x$. (6)

(c) Show that the radius of curvature for the curve

$x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos(\theta/2)$. (6)

4. (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$ (6½)

(b) Find the position and nature of the double points on the curve

$$x^4 + y^3 - 2x^3 + 3y^2 - a^4 = 0$$ (6½)

(c) Trace the curve

$$x^2(x^2 + y^2) = 4(x^2 - y^2).$$ (6½)

5. (a) State and prove Lagrange's Mean Value theorem. (6)

(b) Separate the intervals in which the function

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing. (6)

(c) Prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, for $0 \le x \le 1$. (6)

6. (a) State Cauchy's Mean Value theorem. Give its geometrical interpretation. Also verify Cauchy's mean value theorem for the functions $f(x) = \sin x$, $g(x) = \cos x$ in the interval $\left[-\frac{\pi}{2}, 0\right]$. (6½)

(b) Find the minimum and maximum value of the function x^x. (6½)

(c) If $\lim_{x\to 0} \frac{\sin 3x - a \sin x}{x^3}$ is finite, then find the value of a and the limit. (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 198 **G** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed by the curves $y^2 = 4x$ *and* $y = 2x - 4$. (6)

   (b) Obtain a reduction formula for $\int sec^n x\, dx$, n being a positive integer and hence evaluate $\int sec^6 x\, dx$. (6)

   (c) Evaluate :

$$\int \frac{1}{3sinx - 4cosx} dx \,. \quad (6)$$

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} cos^m x cos\; nx dx = \frac{m}{m+n} \int_0^{\frac{\pi}{2}} \cos^{m-1} x cos(n-1) x dx \,.$$

Further show that

$$\int_0^{\frac{\pi}{2}} \cos^n x\; cosnx\; dx = \frac{\pi}{2^{n+1}} \,,$$

where m and n being positive integers. (6½)

P.T.O.

(b) Find the exact arc length of the curve

$$24xy = y^4 + 48 \textit{ from } y = 2 \textit{ to } y = 4.$$ (6½)

(c) Let V_x and V_y be the volume of the solids that result when the region enclosed by

$$y = \frac{1}{x},\ y = 0,\ x = \frac{1}{2} \text{ and } x = b\ (b > 2)$$

is revolved about x – axis and y – axis, respectively. Is there any value of b for which $V_x = V_y$? (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} log\left(tanx + cotx\right) dx$

(ii) $\int \frac{2x+3}{\sqrt{4x^2+5x+6}}\, dx$ (3+3)

(b) Solve :

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$$ (6)

(c) Given that $y = x + 1$ is a solution of

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6.$$ (6)

(b) Using the concept of Wronskian ,show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \cdot$$

Find the solution $\mathbf{y}(x)$ satisfying the conditions $\mathbf{y}(0) = 2$ and $\mathbf{y}'(0) = -3$. (6)

(c) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ? (6)

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2} + y = tanx\,. \qquad (6\tfrac{1}{2})$$

(b) Solve :

$$a^2\,y^2 z^2\,dx \;+\; b^2\,x^2\,z^2\,dy \;+\; c^2\,x^2\,y^2\,dz \;=\; 0. \qquad (6\tfrac{1}{2})$$

(c) Solve the system of equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$$

$$\frac{dy}{dt} + 5x + 3y = 0 \qquad (6\tfrac{1}{2})$$

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic form :

$$r + 2s + t = 0$$

where $r = \dfrac{\partial^2 z}{\partial x^2}$, $s = \dfrac{\partial^2 z}{\partial x\,\partial y}$, $t = \dfrac{\partial^2 z}{\partial y^2}$. (3)

(ii) Eliminate the arbitrary function f to form the partial differential equation from the following equation

$$z = f\left(\frac{xy}{z}\right). \qquad (3\tfrac{1}{2})$$

(b) Find the general integral of the linear partial differential equation

$$z(xp - yq) = y^2 - x^2. \qquad (6½)$$

(c) Find the complete integral of the partial differential equation

$$p + q = pq. \qquad (6½)$$

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 199 **G** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed between the curves $y = x^2$, $y = \sqrt{x}$; $x = \frac{1}{4}$, $x = 1$. (6)

(b) Evaluate :

$$\int \frac{x^3}{\left(1+x^2\right)^{9/2}}\, dx$$ (6)

(c) If $I_n = \int_0^{\frac{\pi}{2}} \theta \sin^n \theta \; d\theta \;\; (n>1)$, show that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}.$$

Deduce that $I_5 = \dfrac{149}{225}$ (6)

2. (a) Show that

$$\int_0^{\pi/2} Sin^n x\, dx = \frac{n-1}{n} \int_0^{\pi/2} Sin^{n-2} x\, dx \qquad (n \ge 2)$$

Use this result to derive Wallis Sine formulas :

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{\pi}{2}\frac{1.3.5\ \ldots\ldots..(n-1)}{2.4.6\ \ldots\ldots\ldots\ldots n}\ (n \text{ even and} \geq 2)$$

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{2.4.6\ \ldots\ldots.(n-1)}{3.5.7\ \ldots\ldots\ldots.. n}\ (n \text{ odd and} \geq 3) \qquad (6½)$$

(b) Find the volume of the solid that results when the region above x – axis and below ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad (a>0, b>0)$$

is revolved about the x – axis. (6½)

(c) Find the circumference of a circle of radius a from the parametric equation

$x = a\ cos\ t,\ y = a\ sin\ t\ \ (0 \leq t \leq 2\pi).$ (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} log\ sin\ x\, dx$

(ii) $\int \frac{x+2}{\sqrt{4+3x-2x^2}}\, dx$ (3+3)

(b) Solve :

$$(x^2 + y^2 + x)\, dx + xy\, dy = 0. \qquad (6)$$

(c) Given that $y = x$ is a solution of

$$\left(x^2+1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4 \sin \log x \qquad (6)$$

(b) Using the concept of Wronskian, show that e^x and e^{2x} are linearly independent solution of

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 .$$

Find the solution $y(x)$ satisfying the conditions $y(0) = 0$ and $y'(0) = 1$. (6)

(c) Find the orthogonal trajectories of the family of circles

$$x^2 + y^2 = c^2. \qquad (6)$$

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2} + y = \cot x . \qquad (6½)$$

(b) Solve :

$$(yz + z^2)dx - xzdy + xydz = 0 \qquad (6½)$$

(c) Solve the system of equations :

$$\frac{dx}{dt} + \frac{dy}{dt} - y = 2t + 1$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + y = t \qquad (6½)$$

6. (a) (i) Form a partial differential equation by eliminating the constant a and b from the following equation :

$$2z = (ax + y)^2 + b.$$

(ii) Classify the following partial differential equation into hyperbolic, parabolic or elliptic form :

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = n y^{2n-1} \frac{\partial z}{\partial y}. \qquad (3+3½)$$

(b) Find the general integral of the following partial differential equation

$$y^2 p - xy\, q = x(z - 2y) \qquad (6½)$$

(c) Find the complete integral of the partial differential equation

$$(p^2 + q^2) y = qz. \qquad (6½)$$

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2319**

Unique Paper Code : **62354343** **GC-3**

Name of the Paper : **Analytical Geometry and Applied Algebra**

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Identify and sketch the curve :

$$(y - 3)^2 = 6(x - 2)$$

and also label the focus, vertex and directrix. 6

(*b*) Describe the graph of the curve :

$$4x^2 + y^2 + 8x - 10y = -13.$$ 6

(*c*) Sketch the hyperbola :

$$16x^2 - y^2 - 32x - 6y = 57.$$

Also find the vertices, foci, asymptotes and the equation of directrices. 6

2. (*a*) Find the equation of the parabola that has its focus at $(0, -3)$ and directrix $y = 3$. Also state the reflection property of parabola. 6

(*b*) Find an equation for the ellipse with length of minor axis 8 and with vertices $(2, 6)$ and $(2, -4)$ and also sketch it. 6

(*c*) Find and sketch the curve of the hyperbola that has vertices at $(2, 4)$ and $(10, 4)$ and foci are 10 units apart. 6

3. (*a*) Consider the equation :

$$3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0.$$

Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(*b*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 45°$.

(*i*) Find the $x'y'$-coordinate of the point whose xy-coordinates are $\left(\sqrt{2}, \sqrt{2}\right)$.

(*ii*) Find an equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates. 6

(*c*) Find the equation of the sphere through the four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$, $(2, 0, 1)$. 6

4. (*a*) Let $u = i - 3j + 2k$, $v = i + j$ and $w = 2i + 2j - 4k$. Find the length of $3u - 5v + 2w$. Also find the volume of the parallelepiped with adjacent edges u, v and w. 6½

(*b*) Prove that :

$$u.v = \frac{1}{4}\left(\|u + v\|^2 - \|u - v\|^2\right).$$ 6½

(*c*) (*i*) Using vector, find the area of triangle with vertices A(2, 2, 0), B(–1, 0, 2) and C(0, 4, 3).

(*ii*) Sketch the surface $z = \cos x$ in 3-space. 3+3½

5. (*a*) (*i*) Find the parametric equation of line that is tangent to the circle $x^2 + y^2 = 25$ at the point (3, –4).

(*ii*) Show that the lines L_1 and L_2 intersect and find their point of intersection :

$$L_1 : x = 1 + 4t, \quad y - 3 = t, \; z - 1 = 0$$

$$L_2 : x + 13 = 12t, \quad y - 1 = 6t, \; z - 2 = 3t.$$ 3+3½

(*b*) Find the distance between the skew lines :

$$L_1 : x = 1 + 4t, \quad y = 5 - 4t, \; z = -1 + 5t, \; -\infty < t < \infty$$

$$L_2 : x = 2 + 8t, \quad y = 4 - 3t, \; z = 5 + t, \; -\infty < t < \infty.$$ 3+3½

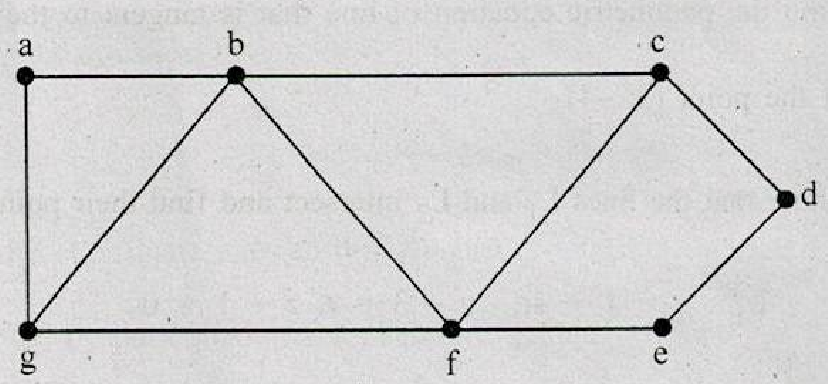
(*c*) (*i*) Find the equation of the plane through (–1, 4, 2) that contains the line of intersection of the planes $4x - y + z - 2 = 0$ and $2x + y - 2z - 3 = 0$.

(*ii*) Do the points (1, 0, –1), (0, 2, 3), (–2, 1, 1) and (4, 2, 3) lie in the same plane. Justify your answer. 3+3½

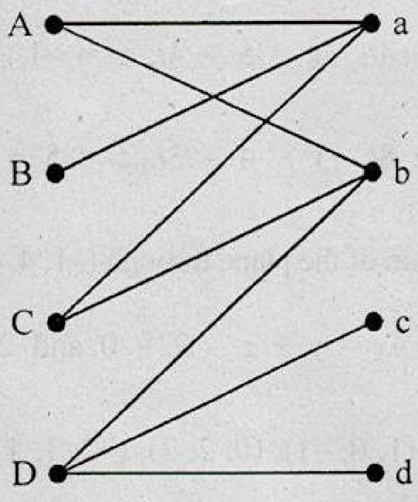
6. (*a*) Define a Latin square. Given an example of a Latin square of order 3. Is it unique ? Justify. 6½

(*b*) Three Pitcher of sizes 7L, 4L and 3L (L = litre) are given. Only 7L pitcher is full. Find a minimum sequence of pouring to make the quantity in three pitchers as 2L, 2L, 3L. 6½

(*c*) (*i*) In the following figure find all sets of two vertices whose removal disconnects the graph :



(*ii*) Find a matching or explain why none exists for the following graph :



3+3½

This question paper contains 3 printed pages. **Your Roll No. ....**

Sl. No. of Ques. Paper	: **2320**	**GC-3**
Unique Paper Code	: **62354343**	
Name of Paper	: **Analytical Geometry and Applied Algebra**	
Name of Course	: **B.A. (Prog.) Mathematics (CBCS)**	
Semester	: **III**	
Duration :	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any two parts from each question.

SET-C

1. (a) Identify and sketch the curve:
$$x = y^2 - 4y + 2$$
and also label the focus, vertex and directrix. 6

(b) Describe the graph of the curve:
$$3(x+2)^2 + 4(y+1)^2 = 12$$
Also find its centre and foci. 6

(c) Describe the graph of the hyperbola:
$$x^2 - y^2 - 4x + 8y - 21 = 0$$
And sketch its graph. 6

2. (a) Find the equation of the parabola that has its vertex at $(1,2)$ and focus at $(4,2)$. Also state the reflection property of parabola. 6

(b) Find the equation of the ellipse whose length of major axis is 26 and foci $(\pm 5,0)$ and also sketch it. 6

(c) Find and sketch the curve of the hyperbola whose foci are $(6,4)$ and $(-4,4)$ and eccentricity is 2. 6

3. (a) Consider the equation:
$$3x^2 + 2xy + 3y^2 = 19.$$

P. T. O.

Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(b) Let an x'y' - coordinate system be obtained by rotating an xy - coordinate system through an angle $\theta = 30°$.

(i) Find the x'y' - coordinate of the point whose xy - coordinates are (2, 4).

(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}xy = 3$ in x'y' - coordinates . 6

(c) Find the equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at $(0,0,7)$. 6

4(a) (i) Find a vector of length 9 and oppositely directed to $\mathbf{v} = -5\mathbf{i} + 4\mathbf{i} + 8\mathbf{k}$.

(ii) Sketch the surface $2x + z = 3$ in 3-space. $3+3\frac{1}{2}$

(b) (i) Find the vector component of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ orthogonal to $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 28\mathbf{k}$.

(ii) Find the area of triangle with vertices P(2, 0, −3), Q(1,4, 5), R(7, 2, 9). $3+3\frac{1}{2}$

(c) Prove that

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

and interpret the result geometrically. $6\frac{1}{2}$

5 (a) Let L_1 and L_2 be the lines whose parametric equations are

L_1 : $x = 4t \quad y = 1-2t \quad z = 2 + 2t$

L_2 : $x = 1 + t \quad y = 1 - t \quad z = -1 + 4t$

(i) Show that the lines L_1 and L_2 intersect at the point (2, 0, 3).

(ii) Find the parametric equation of line that is perpendicular to L_1 and L_2 and passes through their point of intersection. $3+3\frac{1}{2}$

(b) (i) Determine whether the points P_1 (6, 9, 7) , P_2 (9, 2, 0) and P_3 (0, −5, −3) lie on the same line.

(ii) Where does the line

$x = 2 - t, \quad y = 3t, \quad z = -1 + 2t$

intersect the plane $2y + 3z = 6$. $3+3\frac{1}{2}$

(c) (i) Find the equation of the plane through (1, 4, 3) that is perpendicular to the line

$x = 2 + t, \quad y + 3 = 2t, \quad z = -t.$

(ii) Determine whether the planes

$$3x - 2y + z = 1 \ , \ 4x + y - 2z = 4$$

are parallel, perpendicular or neither. $3 + 3\frac{1}{2}$

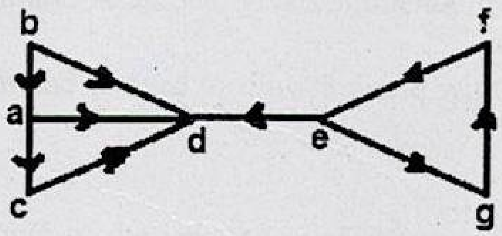
6. (a) Given three containers 3, 7, and 10 liters respectively with the largest being full of water, determine a minimum sequence of pouring method of dividing this quantity of water into two equal amounts of 5 liters using the three containers and no other measuring devices. $6\frac{1}{2}$

(b) Is the following square a Latin square? Can it be a group with the multiplication operation defined?

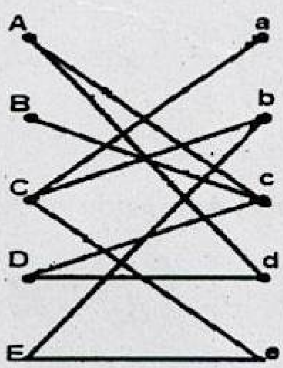
*	1	2	3	4	5
1	1	2	3	4	5
2	2	1	4	5	3
3	3	4	5	2	1
4	4	5	1	3	2
5	5	3	2	1	4

$6\frac{1}{2}$

(c) (i) Given the influence model. Find the sets of minimum number of vertices which can influence every other vertex in the graph.



(ii) Find a matching or explain why none exists for the following graph.



$3 + 3\frac{1}{2}$

1000

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2136**

Unique Paper Code : **32353301** **GC-3**

Name of the Paper : **Latex and HTML**

Name of the Course : **B.Sc. (Hons.) Mathematics—CBCS : Skill Enhancement Course**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(*i*) In LaTeX, items can be listed using .... environment.

(*ii*) Line breaks in a LaTeX document are produced by .... command.

(*iii*) .... command is used to give comments in a latex document.

(*iv*) .... command is used to draw a circle with center (x, y) and radius r in pstricks.

(*v*) .... attribute of the img tag in HTML is used to specify the source of the image.

2. Answer any *eight* parts from the following : 8×2½=20

(*i*) Give the command in LaTeX to obtain the expression $\left(\dfrac{a+b}{x+y}\right)^{\frac{1}{3}}$.

P.T.O.

(*ii*) Write the difference between the commands ***\vdots*** and ***\ddots***.

(*iii*) Write the output of the command :

`$\frac{d}{dx}\left(\int_{0}^{x}f(t)\, dt\right) = f(x)$.`

(*iv*) What is wrong with the following HTML construction :

<p> This is   bold and italics </p> .

(*v*) Give any *three* attributes of the ***font*** tag in HTML.

(*vi*) What is wrong with the following input :

<p> Also checkout the

 University of Delhi </p>

(*vii*) Write a code in LaTeX for typesetting $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

(*viii*) Write a LaTeX code to produce `$\mathbb R$` in the output.

(*ix*) Write the command in LaTeX to generate the expression $x^{x^{x^{x}}}$.

(*x*) What is the output of the command \psarc(1, 1) {2} {0} {70} in pstricks.

3. Answer any *five* questions from the following : 5×5=25

(*i*) Write the code in LaTeX to plot the curves $y = \sqrt{x}$ and $y = x^2$ on the same coordinate. Show the square root function as a dotted curve and the square function as a dashed curve.

(*ii*) Find the errors in the following LaTeX source, write a corrected version and write its output :

```
\Documentclass{article}

\usepackage{amsmath}

\begin{document}

We have following options

\begin{itemize}

\item $$x\ge y$

\item $x\le y$

\item x=y

\end{document}
```

(*iii*) Write a code in LaTeX for typesetting the following expression :

$$\frac{\dfrac{5}{a^2b}-\dfrac{2}{ab^2}}{\dfrac{3}{a^2b^2}+\dfrac{4}{ab}}=\frac{\dfrac{5b-2a}{a^2b^2}}{\dfrac{3+4ab}{a^2b^2}}.$$

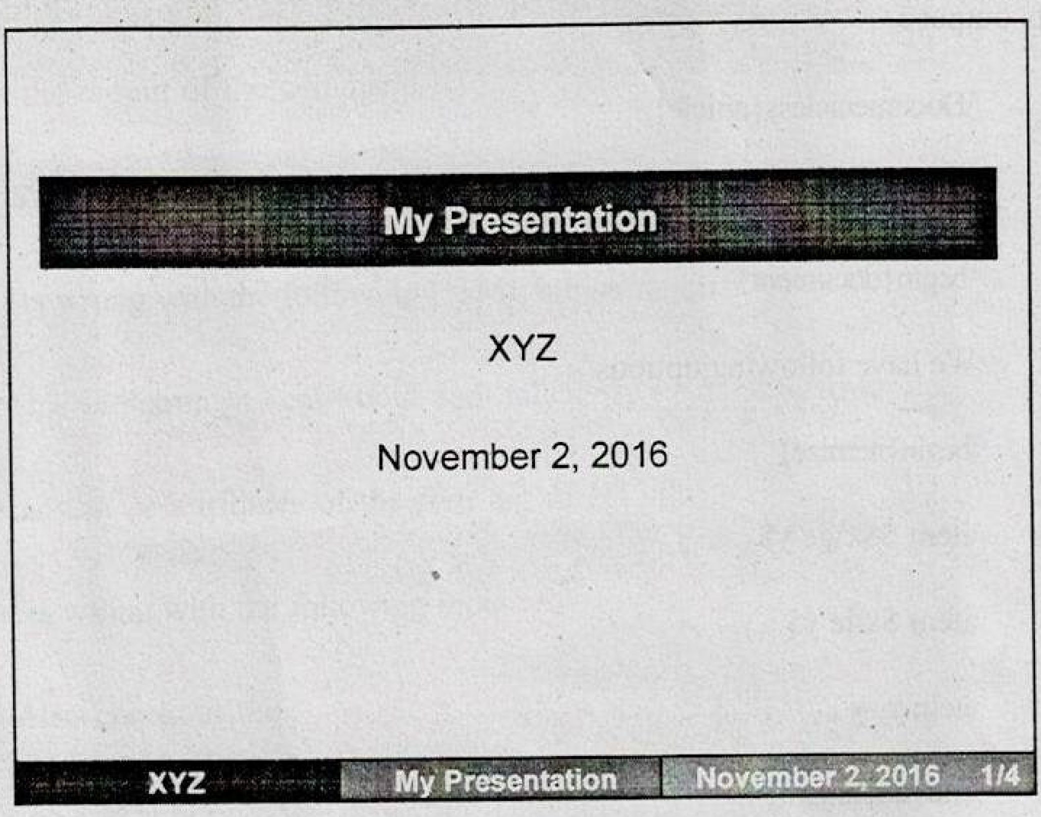
(*iv*) Write a code in LaTeX to typeset the following :

A system of linear equations in 3 variables x_1, x_2 and x_3 can be represented as :

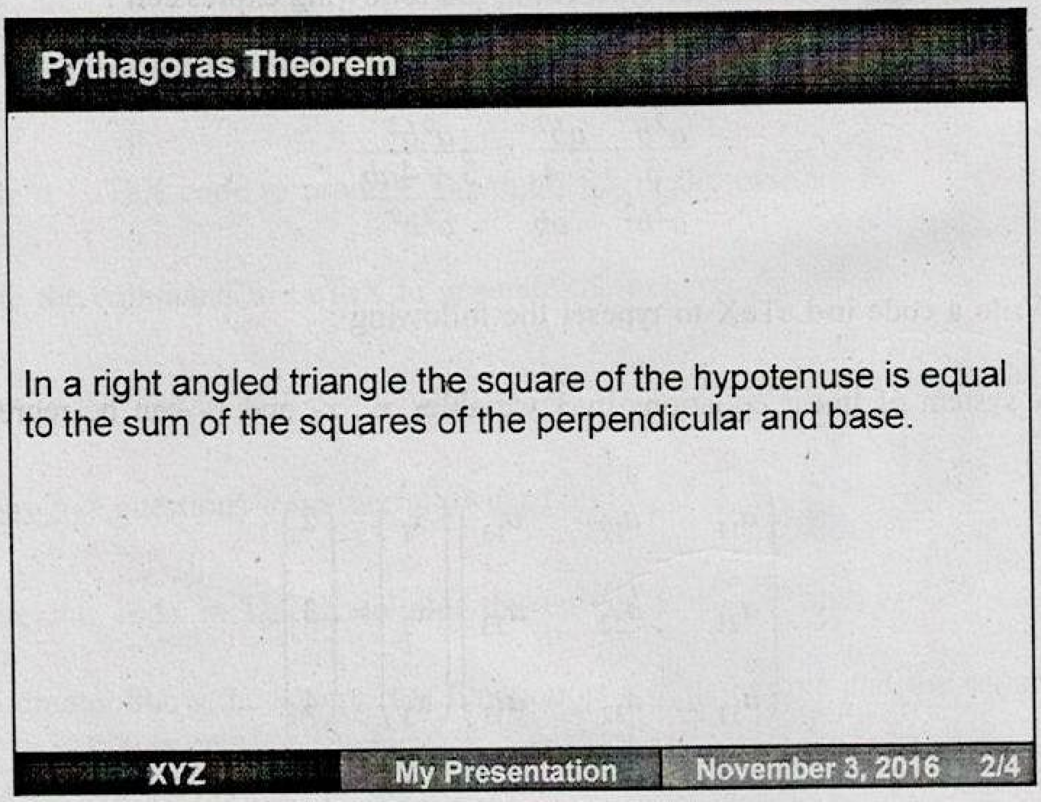
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$

(*v*) Write LaTeX code in beamer to prepare the following presentation :

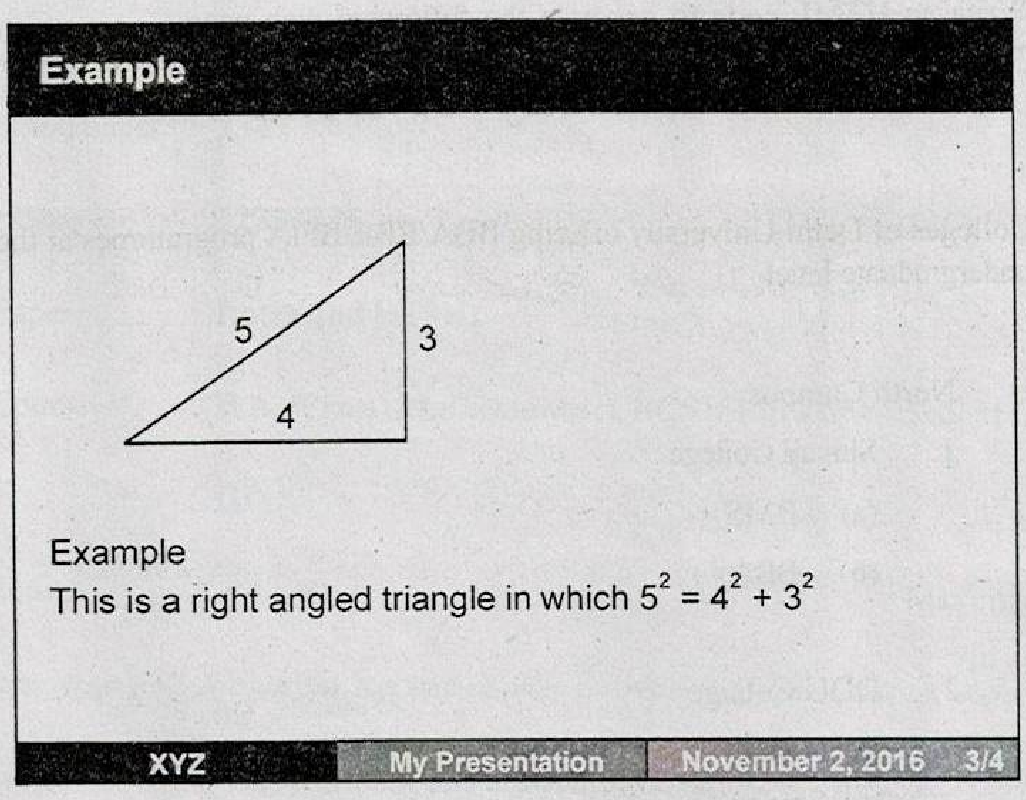
Slide 1



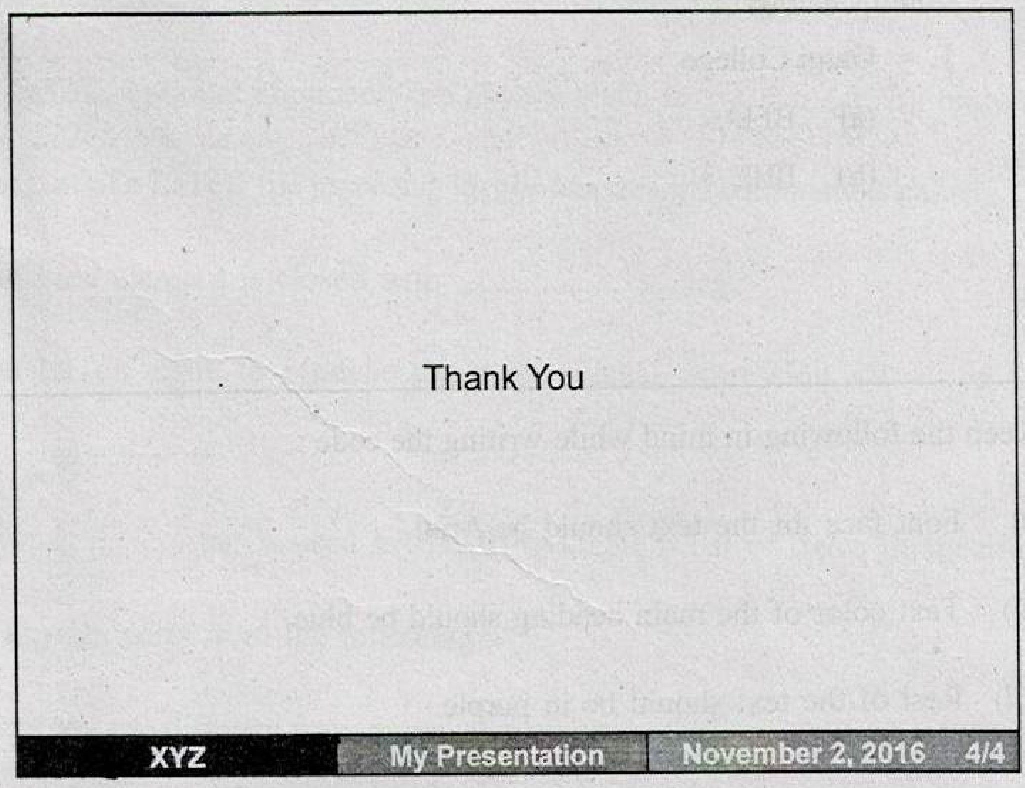
Slide 2



Slide 3



Slide 4



(*vi*) Write an HTML code to generate the following web page :

University of Delhi

Colleges of Delhi University offering BBA/BBE/BFIA programmes at the undergraduate level

- North Campus
  1. Shivaji College
     (a) BMS
     (b) BBA

  2. DDU College
     (a) BBE
     (b) BMS
- South Campus
  1. Gargi College
     (a) BFIA
     (b) BBE

Keep the following in mind while writing the code :

(*i*) Font face for the text should be Arial.

(*ii*) Text color of the main heading should be blue.

(*iii*) Rest of the text should be in purple.

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2349**

Unique Paper Code : **62353325** **GC-3**

Name of the Paper : **Latex and HTML**

Name of the Course : **B.A. (Prog.) Mathematics (CBCS) Skill Enhancement Course**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(*a*) In LaTeX, optional arguments are always given in .... brackets.

(*b*) The part of a LaTeX file preceding ***\begin {document}*** command is called ....

(*c*) The ***html*** element is closed with .... tag.

(*d*) The LaTeX code to produce the mathematical expression $e^{i\theta} = \cos\theta + i\sin\theta$ is ....

(*e*) .... tag is used in HTML to create a list of items in specified order.

2. Answer any *ten* parts from the following : 10×2=20

(1) Write any *two* different ways of including mathematical expressions in LaTeX document.

P.T.O.

(2) Write the difference between the commands ***\ldots*** and ***\cdots***.

(3) What is the output of \pscircle (3, 2){1} in pstricks ?

(4) Write the output of the command **$\sqrt[n]{5}$**.

(5) What is the command for writing the set {0, 1} in LaTeX ?

(6) Explain the difference in the outputs of the following two LaTeX source codes :

(*i*) \begin{document}

Suppose that \$ x = 25 \$

\end{document}

(*ii*) \begin{document}

Suppose that \$\$ x = 25 \$\$

\end{document}

(7) Write a set of commands to be put in the main document in LaTeX to produce :

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

(8) Write the output of :

\documentclass{beamer}

\title{Skill Enhancement Course}

\author{ABC}

\institute{University of Delhi}

```
\begin{document}

\begin{frame}

\titlepage

\end{frame}

\end{document}
```

(9) Write a code in LaTeX to produce the output :

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}.$$

(10) Write the following postfix expressions in the standard form :

x 1 *add* 2 *exp* 1 *x* *sub* *div*.

(11) What is the output of the command **\psline (1, 1) (5, 1) (1, 4) (1, 1)** in pstricks ?

Also, give a rough sketch of the same.

(12) What is wrong with the following input ? What is the right way to do it ?

If $theta = pi$ then $sin theta = 0$.

Answer any *five* parts from the following : 5×5=25

(*i*) Write a code in LaTeX to plot the function :

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ -x^2, & -2 \leq x < 0 \end{cases}.$$

(*ii*) Write the code for the following in LaTeX environment :

Let $x = (x_1, x_2, \ldots\ldots, x_n)$, where the x_i are non-negative real numbers. Set :

$$\mathrm{M}_r(x) = \left(\frac{x_1^r + x_2^r + \ldots\ldots + x_n^r}{n} \right)^{\frac{1}{r}}, \quad r \in \mathbf{R}\backslash\{0\}$$

and

$$\mathrm{M}_0(x) = (x_1 x_2 \ldots\ldots x_n)^{1/n}.$$

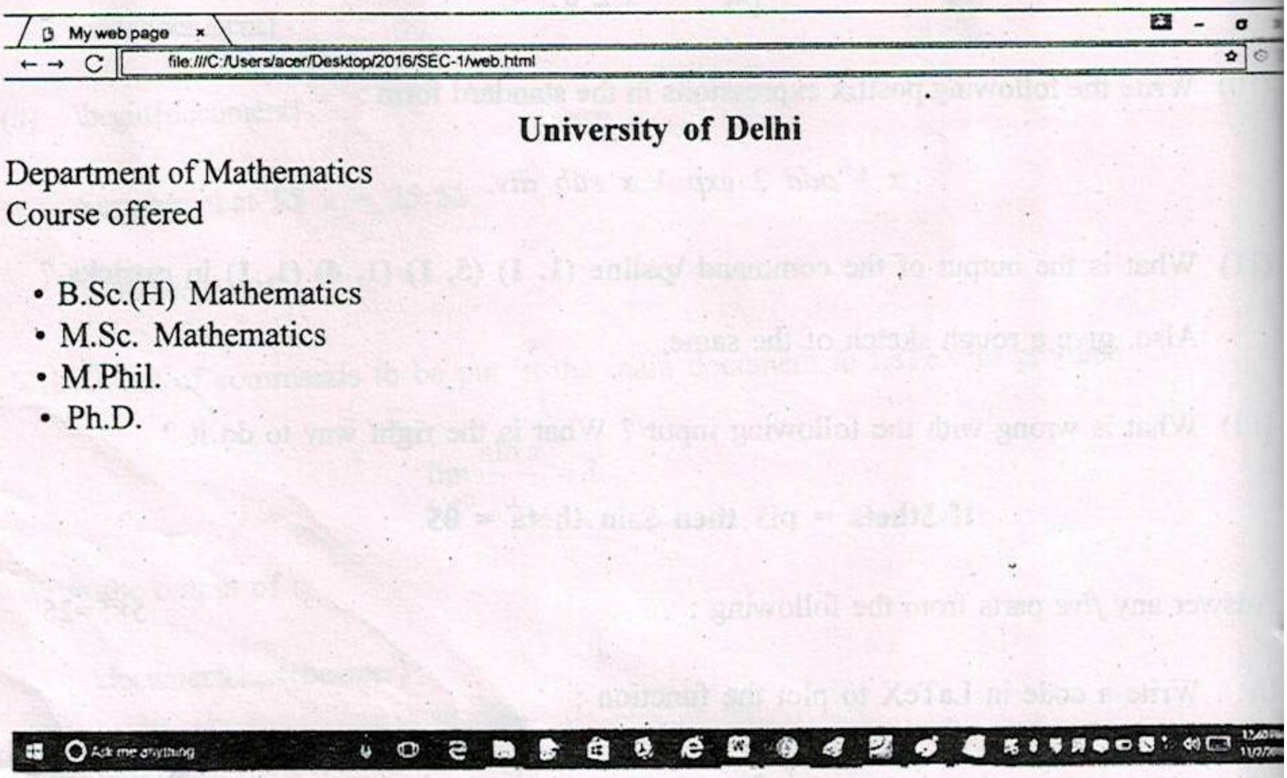
(*iii*) Write a presentation in beamer with the following content :

Slide – 1 contains **the title of the presentation, author's name and affiliation**

Slide – 2 contains the **list of subjects taught in B.A. (Prog.) course**

Slide – 3 contains **Thank You**

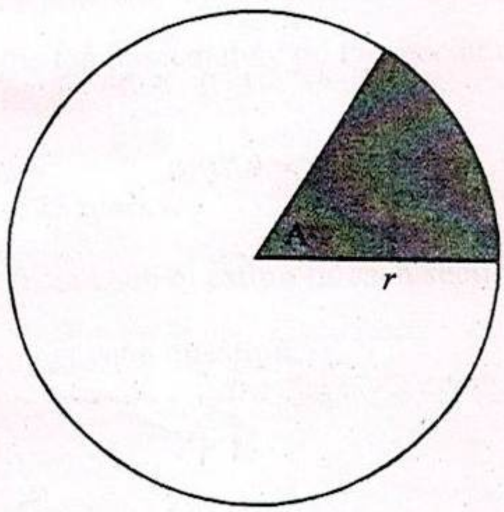
(*iv*) Write an ***html*** code to generate the following web page :



(*v*) Write a code in LaTeX to get the following matrix :

$$A = \begin{bmatrix} a & c & e \\ b & d & f \\ g & i & k \\ h & j & l \end{bmatrix}.$$

(*vi*) Write the code in LaTeX to draw the following circle with shaded sector :



[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 238 **G** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Paper : Analysis

Name of the Course : **B.A. Programme – Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. There are **3** sections.
3. Each section consists of **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Define supremum and infimum of the set $S \subseteq R$. Find the supremum and infimum of the set

$$S = \left\{1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2^2}, \ldots\ldots\ldots, 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots.. + \frac{1}{2^{n-1}}, \ldots\ldots\ldots\right\}$$ (6)

(b) State the properties which make the set R of real umbers, a complete ordered field. (6)

(c) Define limit point of a set. Show that the set N of natural numbers has no limit point. (6)

2. (a) State Bolzano-Weierstrass theorem for sets. Prove that the set $\left\{3^n + \frac{1}{3^n}; n \in N\right\}$ has no limit point. How does it contradict Bolzano-Weierstrass theorem ? (6.5)

(b) Test the continuity of the function

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ at } x = 0$$ (6.5)

(c) Define uniform continuity of a function f on an interval I. Show that the function defined by $f(x) = x^3$ is uniformly continuous on $[-3, 3]$. (6.5)

3. (a) If $a_n \leq b_n \leq c_n$ for all n and $\langle a_n \rangle$ and $\langle c_n \rangle$ converge to l then $\langle b_n \rangle$ also converges to l. (6.5)

(b) State Monotone convergence theorem for sequence and hence prove that the sequence $\langle a_n \rangle$ defined as

$$a_1 = 1,\ a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{1}{(n-1)!}\ (n \geq 2)$$ is convergent. (6.5)

(c) State Cauchy's General Principle of convergence for sequence and show that the sequence $\langle a_n \rangle$ where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$ is not convergent. (6.5)

4. (a) Let $\sum_{1}^{\infty} u_n$ and $\sum_{1}^{\infty} v_n$ be two positive terms series such that $u_n \leq k v_n$ $\forall$ n, k being a fixed positive number then prove that

(i) $\sum u_n$ converges if $\sum v_n$ is convergent

(ii) $\sum v_n$ diverges if $\sum u_n$ is divergent (6)

(b) Test the convergence of the following series :

(i) $\frac{\sqrt{2}-\sqrt{1}}{1} - \frac{\sqrt{3}-\sqrt{2}}{2} + \frac{\sqrt{4}-\sqrt{3}}{3} - \ldots\ldots\ldots$

(ii) $\sum_{1}^{\infty}(-1)^n \frac{n+2}{2^n+5}$

(iii) $\sum_{1}^{\infty}(-1)^{n-1}\frac{1}{n}$ (6)

(c) State Cauchy's General Principle of convergence for an infinite series $\sum_{1}^{\infty} u_n$ and show that the series $\sum_{1}^{\infty}\frac{1}{n}$ does not converge. (6)

5. (a) A bounded function f is integrable on [a, b] iff for every $\varepsilon > 0$, there exists partition P of [a, b] such that $U(P, f) - L(P, f) < \varepsilon$. (6.5)

(b) Show that $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ converges. (6.5)

(c) Show that $\int_0^2 x^4\left(8-x^2\right)^{-\frac{1}{2}}dx = \frac{16}{3}\beta\left(\frac{5}{3}, \frac{2}{3}\right)$. (6.5)

6. (a) Find the Fourier series of the function f where

$$f(x) = \begin{cases} -1, & \text{for } -\pi \le x < 0 \\ 1, & \text{for } 0 \le x \le \pi \end{cases}$$ (6)

(b) State Cauchy's uniform convergence criteria for a sequence of functions.

Test the sequence $\langle f_n(x) \rangle$ for uniform convergence, where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}}, \quad 0 \le x \le 2\pi \qquad (6)$$

(c) (i) Find the radius of convergence of the power series

$$\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \ldots\ldots\ldots$$

(ii) If f is defined in [0, 1] by the condition

$f(x) = (-1)^{r-1}$, when $\frac{1}{r+1} < x \le \frac{1}{r}$, $(r = 1, 2, 3, \ldots\ldots)$,

$f(0) = 0$. Show that $\int_0^1 f(x)\,dx = \log 4 - 1$ (6)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 239 **G** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Paper : Analysis

Name of the Course : **B.A. Programme – Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. There are **3** sections.
3. Each section consists of **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Let A and B be non empty subsets of R and let

$$C = \{x + y: x \in A, y \in B\}$$

If each of the sets A and B has a supremum, show that C has a supremum and Sup C = Sup A + Sup B. (6)

(b) Give example of a set $S \subseteq R$ which has

(i) Exactly one limit point

(ii) Infinite number of limit points

(iii) Two limit points (6)

(c) Show that function defined as $f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$

is continuous only at $x = 0$. (6)

2. (a) Define an open set $S \subseteq R$. Show that intersection of two open sets is again an open set but the intersection of infinite number of open sets need not be open. Justify your answer by an example (6.5)

(b) Show that the function $f(x) = \lim_{n\to\infty} \frac{x^n}{1+x^n}$ for all $x \geq 0$,

is continuous at all points except at $x = 1$. (6.5)

(c) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2, 2]$. (6.5)

SECTION II

3. (a) Show that $\lim_{n\to\infty} (n)^{1/n} = 1$. (6.5)

(b) If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences such that

$\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$, then show that

$$\lim_{n\to\infty} (a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right) = ab.$$ (6.5)

(c) If $\lim_{n\to\infty} a_n = l$, then show that $\lim_{n\to\infty} \frac{a_1 + a_2 + \ldots\ldots + a_n}{n} = l$. (6.5)

4. (a) State and prove D'Alembert's Ratio Test for a positive terms series $\sum_{1}^{\infty} u_n$.

(6)

(b) Test the convergence of the following series :

(i) $\frac{x}{1.3}+\frac{x^2}{2.4}+\frac{x^3}{3.5}+\frac{x^4}{4.6}+\ldots\ldots$ for all values of x.

(ii) $\sum_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(iii) $\sum_{1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ (6)

(c) Define absolute convergence and conditional convergence of an infinite series. Show that every absolutely convergent series is convergent. With the help of an example, show that the converse need. (6)

SECTION III

5. (a) Define Riemann integrability of a bounded function over [a, b]. Show that every monotonic function on [a, b] is integrable on [a, b]. (6)

(b) Discuss the convergence of the improper integral $\int_{0}^{\frac{\pi}{2}} \log \sin x \, dx$. (6)

(c) (i) Define Beta and Gamma functions. What is the relation between Beta and Gamma functions ?

(ii) Show that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$ (6)

6. (a) Find the Fourier series in $[-\pi, \pi]$ for the function :

$$f(x) = \begin{cases} x, & \text{if } -\pi < x \le 0 \\ 2x, & \text{if } 0 < x \le \pi \end{cases} \quad (6.5)$$

(b) State Weierstrass's M-test for the series of the functions f_n defined on [a, b]. Show that the series $\sum_{1}^{\infty} \frac{\sin nx}{n^p}$ is uniformly convergent for all real values of x if p > 1. (6.5)

(c) (i) Find the radius of convergence of the power series

$$\sum_{0}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n .$$

(ii) Discuss the Riemann integrability of the function f on [0, 3] where f(x) = [x], [x] is the greatest integer ≤ x. (6.5)

'5 Gp R

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2257 **GC-3** **Your Roll No...............**

Unique Paper Code : 32355101

Name of the Paper : GE – I Calculus

Name of the Course : **Generic Elective for Hons. Courses**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION I

1. Use $\varepsilon - \delta$ definition to show that

$$\lim_{x \to 3}(3x-7) = 2.$$

2. Find the equations of the asymptotes for the curve

$$f(x) = \frac{x^3+1}{x^2}.$$

3. Find the Linearization of

$$f(x) = \sin x \quad \text{at} \quad x = \pi.$$

4. For $f(x) = x^3 - 3x + 3$

   (i) Identify where the extrema of 'f' occur.

(ii) Find where the graph is concave up and where it is concave down.

5. Use L'Hôpital's rule to find

$$\lim_{x \to \pi/2} \frac{1-\sin x}{1+\cos 2x}.$$

6. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

7. Find the length of the astroid

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi.$$

SECTION II

8. State Limit comparison test. Using the limit comparison test, discuss the convergence of

$$\int_1^\infty \frac{dx}{1+x^2}.$$

9. Identify the symmetries of the curve and then sketch the graph of

$$r = \sin 2\theta.$$

10. Solve the initial value problem for $\vec{r}$ as a vector function of t

Differential equation : $\frac{d^2\vec{r}}{dt^2} = 32\hat{k}$

Initial Conditions : $\vec{r}(0) = 100\hat{k}$

and : $\left(\frac{d\vec{r}}{dt}\right)_{t=0} = 8\hat{i} + 8\hat{j}$

11. Find the curvature for the helix

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\ a, b \geq 0\quad a^2 + b^2 \neq 0$$

12. Write the acceleration vector $\vec{a} = a_T\hat{T} + a_N\hat{N}$ at the given value of t without finding $\hat{T}$ and $\hat{N}$ for the position vector given by

$$\vec{r}(t) = (t\cos t)\hat{i} + (t\sin t)\hat{j} + t^2\hat{k},\quad t = 0$$

13. Show that $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0\quad , & (x,y) = (0,0) \end{cases}$

is continuous at every point except at the origin.

14. If $f(x,y) = \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$

(i) Find the domain of the given function f(x,y).

(ii) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

SECTION III

15. If $z = 5\tan^{-1} x$ and $x = e^u + \ln v$,

find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ using chain rule, when $u = \ln 2$, $v = 1$.

16. Find the directions in which the given function f increase and decrease most rapidly at the given point p_0. Then, find the derivative of the function in those directions.

$$f(x,y,z) = \frac{x}{y} - yz,\ p_0(4,1,1)$$

17. Find parametric equations for the line tangent to the curve of intersection of the given surfaces at the given point.

$$\text{Surfaces}: x + y^2 + 2z = 4, \quad x = 1$$

$$\text{Point}: (1, 1, 1).$$

18. Find equations for the

(a) Tangent plane and

(b) Normal line at the point p_0 on the given surface

$$z^2 - 2x^2 - 2y^2 - 12 = 0; \quad p_0(1, -1, 4).$$

19. Find the absolute maxima and minima of the function $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.

20. If $f(x,y) = x\cos y + ye^x$, find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y\, \partial x}, \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x\, \partial y}$.

21. If $f(x,y) = x - y$ and $g(x,y) = 3y$

Show that

(i) $\nabla(fg) = g\nabla f + f\nabla g$

(ii) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$

12/13 30

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2594 **GC-3** **Your Roll No..............**

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : **Generic Elective – 3 for Hons. Courses, Under CBCS**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Find an integrating factor and solve the differential equation :

$$\left(e^{x+y} - y\right)dx + \left(xe^{x+y} + 1\right)dy = 0. \qquad (6.5)$$

(b) Solve the equation : $y' + (x + 1)y = e^{x^2} y^3$, $y(0) = 0.5$. (6.5)

(c) Find the orthogonal trajectories of the family of parabolas $y = ce^{-3x}$. (6.5)

2. (a) Show that e^{3x} and xe^{3x} form a basis of the following differential equation $y'' - 6y + 9y - 0$. Find also the solution that satisfies the conditions $y(0) = -1.4$, $y'(0) = 4.6$. (6)

(b) Solve the initial value problem : (6)

$$x^2y'' + 3xy' + y = 0,\ y(1) = 4,\ y'(1) = -2.$$

(c) Find the radius of convergence of the series : $\sum_{m=2}^{\infty} \frac{(-1)^m (x-1)^{2m}}{4^m}$ (6)

3. (a) Find a general solution of the following nonhomogeneous differential equation :

$$y'' + 3y' + 2y = 30e^{2x}$$

using variation of parameters. (6.5)

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation : $y'' - 4y' + 4y = 2e^{2x}$. (6.5)

(c) Find a homogeneous linear ordinary differential equation for which two functions x^3 and x^{-2} are solutions. Show also linear independence by considering their Wronskian. (6.5)

4. (a) Find the general solution of the partial differential equation

$$yzu_x - xzu_y + xy(x^2 + y^2)u_z = 0.$$ (6)

(b) Find a general solution of the differential equation :

$$(x^2D^2 + xD - 4I)y = 0, \text{ where } D = \frac{d}{dx}.$$ (6)

(c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = -5y_1 + 2y_2, \ y_1(0) = 1$$

$$\frac{dy_2}{dt} = 2y_1 - 2y_2, \ y_2(0) = -2 \tag{6}$$

5. (a) Find a power series solution of the following differential equation, in powers of x

$$y'' + xy' - 2y = 0. \tag{6.5}$$

(b) Find the solution of the quasi-linear partial differential equation :

$$u(x + y)u_x + u[x - y)u_y = x^2 + y^2$$

with the Cauchy data $u = 0$ on $y = 2x$. (6.5)

(c) Reduce the equation : $yu_x + u_y = x$

to canonical form, and obtain the general solution. (6.5)

6. (a) Solve the initial-value problem :

$$u_x + 2u_y = 0, \quad \mu(0, y) = 4e^{-2y}$$

using the method of separation of variables. (6)

(b) Obtain the canonical form of the equation : $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$, and hence find the general solution. (6)

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0.$$

into canonical form. (6)